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Modelling the Composition of Personal Sector Wealth in the UK

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Modelling the composition of personal sector wealth in the UK

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The allocation of UK personal sector wealth across five broad asset categories (net financial wealth, housing (and durable assets) wealth, state pension wealth, private pension wealth, and human capital) is investigated using the FAIDS (financial AIDS) model. Apart from total wealth and returns, additional variables relating to capital market imperfections, and demographic, labour market and cross-sector spillover effects turn out to be significant. The adjustment of portfolio weights to shocks is very slow, taking up to 21 years for some asset categories.

I. INTRODUCTION

This study investigates the factors determining the allocation of aggregate personal sector wealth in the UK across five broad asset categories: net financial wealth, housing (and durable assets) wealth, state pension wealth, private pension wealth, and human capital,¹ using a version of Deaton and Muellbauer's (1980) AIDS model that has been developed for modelling financial assets, the FAIDS model.^{2,3}

The portfolio weights of the five asset categories for the period 1948–1994 are shown in Figs. 1–3. The weight in net financial assets (F in Fig. 1) fell steadily during the 1950s and 1960s from a high point of 12% to around 3%, at

which level it remained throughout the high-inflation 1970s and 1980s, before rising to above 4% in the 1990s as inflation subsided. The weight in housing (and durable assets) wealth (H) has been on a rising trend since the late 1950s, and the impact of the three housing booms of the early and late 1970s and late 1980s is clearly discernible. Figure 2 shows the growing importance in total wealth of the value of accrued pension rights. The share of state pension wealth (S) in 1948 was about 2.5%, but it rose steadily if unevenly until the mid-1970s,⁴ after which it flattened out at about 6% of total wealth. The growth rate in the share of private pension wealth (P) has been even greater than this, with the share rising over the period from less than 1% to 6%: the effect of the growth in membership

¹ The data for the first four categories were constructed in Blake and Orszag (1999), while the data on human capital and the returns on all the asset categories are constructed in the appendix to this paper.

² This model has been used by Barr and Cuthbertson (1991a, 1991b, 1994) to study the demand for financial assets by different groups of investors and by Dinenis and Scott (1993) to examine UK pension funds' portfolio composition.

³ The original intention of this study was to estimate portfolio share equations derived from Merton's (1969, 1971) continuous time model, since the consumption equation from Merton's model had been successfully estimated in a companion study (Blake (forthcoming)). However, the coefficients of the model are complex functions of preference parameters and the first two moments of the asset return distribution and these could not be identified separately. Further, Sargan's likelihood test (see Pesaran and Pesaran (1997)) favoured the AIDS model (the difference in likelihoods was 3.5). There are other reasons for rejecting the Merton model in favour of the AIDS model: estimates of long-run elasticities were implausibly high, and there was evidence of serial correlation and predictive failure in most of the equations.

⁴ The jagged nature of the rise is explained by the fact that the value of the basic state pension was uprated only periodically during this period.

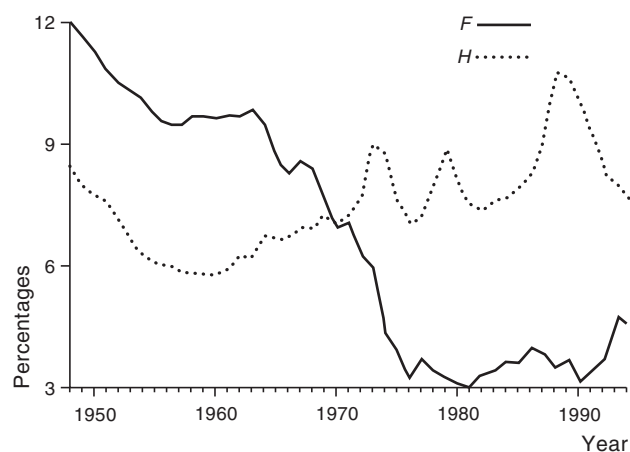


Fig. 1. Portfolio weights of net financial wealth and housing (and durable assets) wealth (percentages), 1948–1994

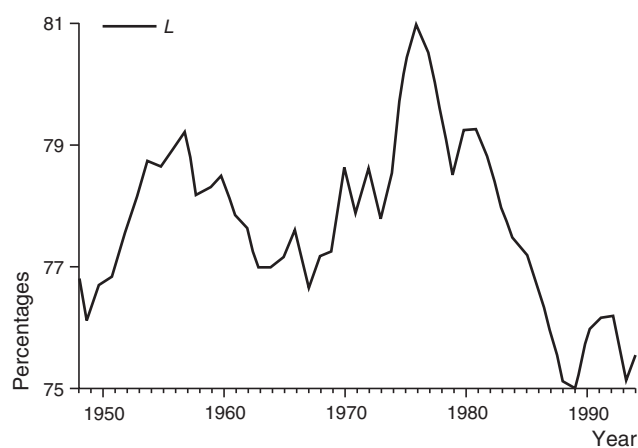


Fig. 3. Portfolio weight of human capital (percentages), 1948–1994

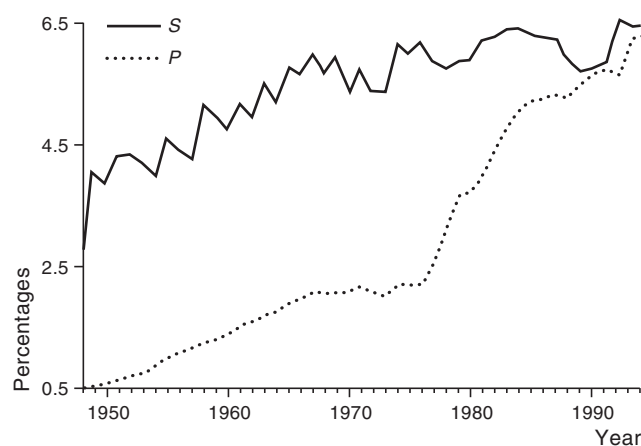


Fig. 2. Portfolio weights of state and private pension wealth (percentages), 1948–1994

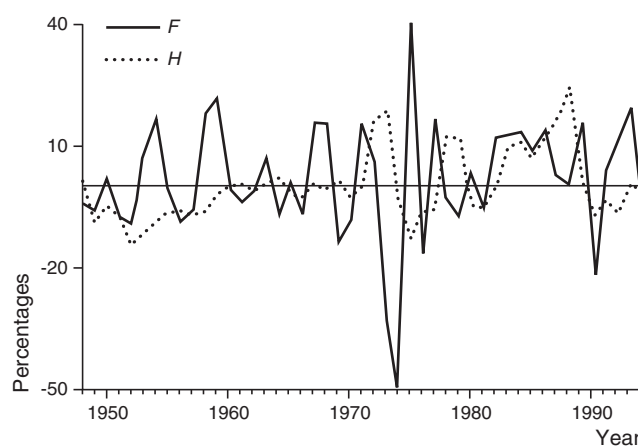


Fig. 4. Real returns on net financial wealth and housing (and durable assets) wealth (per cent per annum), 1948–1994

in these schemes from the mid 70s is particularly noticeable. The combined weight of these four asset categories never amounted to more than 25% of total personal wealth over the sample period. Figure 3 explains why. The share of human capital (the expected discounted value of career earnings, L) has never fallen below 75% of total wealth throughout the post-war period. It reached a peak of 81% in 1976 (largely explained by the collapse of the London stock market two years earlier), but subsequently fell steadily to around 75% by the end of the period.

Figures 4–5 shows the real returns on these asset categories. The average annual real return on net financial wealth was 1.6% with a standard deviation of 14.7% (see Table A4). The extent of this volatility is clearly observable from Figure 4 (see F) especially around the time of the stock market collapse and recovery in 1974–1975. The average real return on housing (and durable assets) wealth was 0.2% with a volatility of 8.3%. The depreciation on

durable assets means that the return on durable assets is always negative and this helps to bring down the average return for this combined asset category: the average real return on housing itself was very high over the sample at 6%. Figure 5 shows that real returns available on state pensions (S) were both very high and stable over the period, with an average return of 10.5% and a standard deviation of just 0.8%. The real returns on private pension schemes (P) were also very high at 9.1%, but their volatility at 2.4% was higher because the defined contribution component of this category involves investments such as equities that are similar to those of net financial assets, although the defined benefit component which is linked to more stable earnings growth helps to attenuate the volatility. The high real returns to membership of UK pension schemes arises from a combination of tax relief on both contributions and investment returns and from the fact that a proportion of the total contributions is paid by the

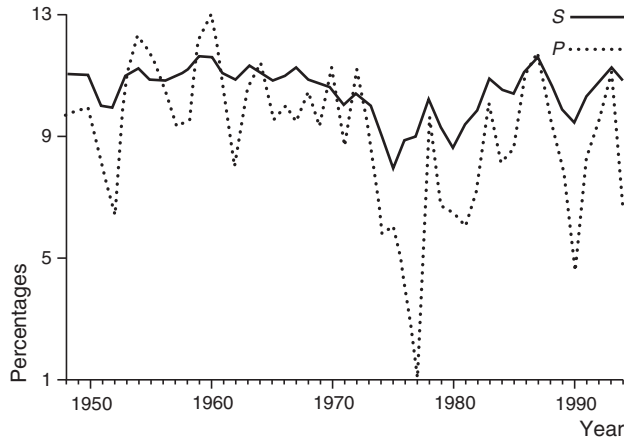


Fig. 5. Real returns on state and private pension wealth (per cent per annum), 1948–1994

employer. Human capital was constructed using an assumed real discount rate of 3% p.a.⁵

What explains the changes in portfolio weights over the post-war period? Do the high, stable returns on housing wealth explain the shift away from financial assets or are wealth effects more important? To what extent are financial assets and housing complements? To what extent are housing and pension wealth substitutes during an individual's retirement phase? Do liquidity constraints and other capital market imperfections affect the asset allocation? Do other variables such as demographic factors, labour market status, and spillovers from other sectors of the economy have a significant impact? Some asset categories involve mandatory participation (e.g., the basic state pension scheme) or contractual obligations (e.g., once someone has joined an occupational pension scheme) or are very slow to adjust over time (e.g., human capital). Do these restrictions on disposability limit the ability of individuals to alter their holdings in other asset categories in order to achieve the desired portfolio weights in all categories? An attempt is made to address questions of this kind in the remainder of this paper. Section II develops the theoretical models, while Section III discusses the empirical results. Section IV concludes, and an explanation of how the data were constructed is given in the appendix.

II. THE FAIDS MODEL OF PORTFOLIO COMPOSITION

In the FAIDS model, the objective of a representative agent is to:⁶

$$\text{Max } \bar{U}(\theta_{1t}W_t, \dots, \theta_{Nt}W_t) \quad (1)$$

subject to a budget constraint:

$$\bar{W}_{t+1} = \sum_{i=1}^N (1 + \bar{r}_{it}) \theta_{it} W_t \quad (2)$$

where bars over variables indicate expected values and where:

- $U(\cdot)$ – utility function,
- W_t – real wealth at time t ,
- θ_{it} – weight in the portfolio of the i th asset category at time t ,
- r_{it} – real return on the i th asset category at time t ,
- N – number of asset categories in the portfolio.

However, rather than maximize the expected utility function in Equation 1, Deaton and Muellbauer suggest minimizing the associated cost function. Using a PIGLOG functional form for this cost function leads to optimal (long-run) portfolio weights of the form:⁷

$$\theta_{it}^* = a_i^* + b_i^* \ln W_t + b_i^* \ln(1 + \bar{r}_{Wt}) + \sum_{j=1}^N c_{ij}^* \ln(1 + \bar{r}_{jt}) \quad (3)$$

where r_{Wt} is an index measure of the total return on assets defined by:

$$\ln(1 + \bar{r}_{Wt}) = a_0^* + \sum_j^N a_j^* \ln(1 + \bar{r}_{jt}) + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N c_{ij}^* \ln(1 + \bar{r}_{it}) \ln(1 + \bar{r}_{jt}) \quad (4)$$

Given the low order of magnitude of \bar{r}_{it} , the cross-product term in Equation 4 will be negligible and can be dropped. In addition, by setting $a_0^* = 0$ and $a_j^* = \theta_j$, then \bar{r}_{Wt} is the expected return equivalent of the Stone index, i.e., the value-weighted average expected return on assets held in the portfolio.

⁵ This is the discount rate assumed by the Government Actuary's Department in its calculations of the accrued value of state pension rights.

⁶ The utility function Equation 1 is time-separable and the moments of the distribution functions generating asset real returns in Equation 2 are assumed to be time-invariant. These assumptions are necessary to derive a tractable FAIDS model (see, e.g., Dinienis and Scott, 1993). The validity of the assumption of time-invariant moments is assessed below.

⁷ For a derivation of Equation 3, see, e.g., Barr and Cuthbertson (1991a) or Dinienis and Scott (1993). Note, however, that these authors, in order to preserve the original AIDS specification, work with the log expected prices of financial assets, which are defined as $\ln \bar{P}_{jt} = \ln(1/(1 + \bar{r}_{jt})) = -\ln(1 + \bar{r}_{jt})$, whereas Equation 3 works directly with the log expected returns. The coefficients in Equations 3 and 4 should be interpreted accordingly. Further note that the second and higher moments of the asset return distributions are subsumed in the intercepts of Equations 3 and 4; the validity of this restriction is tested below.

The standard FAIDS model predicts that the optimal portfolio weights are linear in the logarithms of total wealth and expected real asset returns, with no other variables predicted to have any significance. In practice, however, the following possibilities must be allowed for. Since the capital markets are not perfect, individuals may be liquidity-constrained and these constraints may change over time as a result of, say, financial deregulation. Other variables, apart from wealth and asset expected returns may influence the portfolio weights. Finally, individuals are unlikely to be holding optimal portfolios at all times, and there will be costs both of adjusting actual portfolios towards optimal portfolios and of being away from optimal portfolios.

These possibilities are accounted for in the following ways:

- *Income effects.* The effect of liquidity constraints is to introduce current income into the portfolio shares equation, just as it does in the consumption function (Flavin, 1985; Zeldes, 1989). Some investigators have included the standard deviation of current income (*YVOL*), since uncertainty about income can reduce consumption and increase precautionary asset holdings (Skinner, 1988; Caballero 1990; Hendry, 1994). Following Hendry (1994, Equation 8), *YVOL* is estimated as the absolute value of the residuals in the following regression equation:

$$\begin{aligned} \Delta \ln Y_t = & 0.0214 + 0.4731 \Delta \ln Y_{t-1} - 0.3117 \Delta \ln P_t \\ & (2.52) \quad (2.74) \quad (2.15) \\ & + 0.1717 \Delta \ln P_{t-1} - 0.2260 \ln YD_{t-1} \\ & (1.10) \quad (2.13) \end{aligned}$$

$\bar{R}^2 = 0.39$, $DW = 1.80$, serial correlation $\chi^2(1) = 1.99$, functional form $\chi^2(1) = 1.74$, normality $\chi^2(2) = 10.58$, heteroscedasticity $\chi^2(1) = 1.77$, where Y is real income, P is the price level and $\ln YD$ is the deviation of $\ln Y$ from a linear deterministic trend. Others have included the inflation rate, because 'nominal rather than real interest rate payments are considered to be income in the national accounts, hence in inflationary times consumers are forced to increase saving simply to keep their debt position stable' (Bayoumi, 1993b, p. 1434). Hendry and von Ungern-Sternberg (1981) and Hendry (1994) use a variable that results from multiplying the value of liquid assets by the inflation rate (\dot{P}_F). A high value for \dot{P}_F could have the following effects: it might induce individuals to increase their savings in order to maintain the real value of their liquid assets, it might encourage them to increase current consumption to avoid paying higher prices in the future, or it might induce them to switch into assets that are better inflation hedges. The first two effects

influence total wealth as well as its composition, while the third influences only the composition.

- *Financial deregulation.* The late 1970s and 1980s was a period of substantial financial deregulation and increasing competition between financial institutions. The effect of financial deregulation on the relaxation of liquidity and capital market constraints has been investigated by a number of authors, e.g. Manchester and Poterba (1989), Bayoumi and Koujianou (1991), Campbell and Mankiw (1991), Miles (1992), and Bayoumi (1993a, 1993b). Most of these studies concluded that liquidity constraints had an important impact on consumption in the 1970s but, by the end of the 1980s, this impact had largely vanished except in the case of Japan. There have been no similar studies of the impact of financial deregulation on the broad asset allocation of the personal sector. As a proxy for financial deregulation, Bayoumi (1993a, 1993b) used the ratio of total outstanding consumer credit to GDP, transformed to equal 0 in 1975 and 1 in 1988 (*FINDREG*) on the grounds that 'since consumer credit is used to finance deviations of consumption from income, this ratio is a useful measure of the extent to which consumers are using credit markets to smooth consumption' (Bayoumi, 1993b, p. 1435).⁸
- *Life cycle factors.* The system (Equation 3) explains the optimal portfolio behaviour of an infinitely-lived representative agent with no bequest motive. In this framework, the portfolio composition is independent of the individual's age. Different investigators have accounted for life cycle factors in a variety of ways. Some include the proportions of the population who are respectively young (the youth dependency ratio (*YOUTHDR*)) and old (the elderly dependency ratio (*AGEDR*)) (e.g. Modigliani, 1970; Feldstein, 1980 and Miles and Patel, 1996). Others include life expectancy (*LIFEXP*) (e.g. Hamermesh, 1985).
- *Labour market status.* Clearly an individual's labour market status (employed or unemployed, in work or retired) can affect asset allocation. Campbell and Mankiw (1991), for example, test whether the increase in unemployment (*UN*) in the 1980s might have tended to counteract the positive impact of financial deregulation. However, not everyone who is not in work is registered as unemployed. An alternative measure that can be used is the labour force participation rate (*LABPR*). Other investigators have included either the retirement age (e.g. Munnell, 1974; Crawford and Lilien, 1981), or the labour force participation rate of the elderly (*AGEPR*) (Feldstein, 1980).
- *Spillover effects from other sectors.* Some investigators have included the savings of the corporate (*SC*) and

⁸ Other measures have been used such as the index of financial deregulation developed by Muellbauer and Murphy (1993).

government (SG) sectors since these might be substitutes for personal sector asset holdings (Feldstein, 1974 who uses corporate retained earnings), Barro, 1974, 1978 (who uses the government surplus); Bayoumi, 1995; Miles and Patel, 1996. Another possibility is to include the surplus in occupational pension schemes (SURPLUS), on the grounds that some of the surplus might be shared with pensioners, but in any event increases the wealth of the shareholders of companies running surpluses.

- *Asset return volatilities.* Second-moment or risk terms, because they are subsumed in the constant terms, have a ‘fixed effect’ in the FAIDS model. To test the validity of this, the separate standard deviations of asset returns was included, approximated by the absolute value of the residuals from first-order autoregressive processes for asset returns.⁹

If these additional M variables are denoted by Z_{jt} , the long-run FAIDS model (Equation 3) becomes:

$$\theta_{it}^* = a_i^* + b_i^* \ln(W_t(1 + \bar{r}_{Wt})) + \sum_{j=1}^N c_{ij}^* \ln(1 + \bar{r}_{jt}) + \sum_{j=1}^M h_{ij}^* Z_{jt} \quad (5)$$

To account for dynamic adjustment Equation 5 is expressed in matrix notation:

$$\theta_t^* = \Pi^* \mathbf{x}_t \quad (6)$$

where θ_t^* is the $N \times 1$ vector of optimal portfolio weights, \mathbf{x}_t is the $(N + M + 2) \times 1$ vector of explanatory variables and Π^* is a conformable matrix of long-run coefficients. It is assumed that the representative individual will choose actual portfolio weights to minimize the following quadratic cost function,¹⁰ subject to the additivity constraint $\iota' \theta_t = 1$, where ι is the unit vector:

$$\text{Min}_{\theta_t} \frac{1}{2} \left\{ (\theta_t - \theta_{t-1})' \Psi (\theta_t - \theta_{t-1}) + (\theta_t - \theta_t^*)' \Omega (\theta_t - \theta_t^*) \right\} \quad (7)$$

The first term represents the cost of adjusting actual portfolio weights over time, while the second term represents the cost of actual weights deviating from optimal weights. The solution to Equation 7 is the partial adjustment model:¹¹

$$\widehat{\theta}_t = \widehat{\Lambda} \widehat{\theta}_t^* - (\mathbf{I} - \widehat{\Lambda}) \widehat{\theta}_{t-1} \quad (8)$$

$$= \widehat{\theta}_t^* - \widehat{\Lambda}^{-1} (\mathbf{I} - \widehat{\Lambda}) \Delta \widehat{\theta}_t \quad (9)$$

where Δ is the difference operator $(1-L)$, \mathbf{I} is the $((N-1) \times (N-1))$ identity matrix, and $\widehat{\Lambda}$ is the following matrix Λ of adjustment coefficients with the last (N th) row and column deleted:

$$\Lambda = \Phi - \frac{\Phi \Omega^{-1} \iota' \Phi}{\iota' \Phi \Omega^{-1} \iota} \quad (10)$$

and

$$\Phi = (\Omega + \Psi)^{-1} \Omega \quad (11)$$

The hatted vectors in Equation 8 also indicate the removal of their last row in order to avoid the singularity discussed by Anderson and Blundell (1982): since $\iota'(\theta_{t-1} - \theta_{t-1}^*) = 0$, only $(N-1)$ disequilibrium shares are needed in Equation 8. Substituting $\widehat{\Pi}^* \mathbf{x}_t$ (where $\widehat{\Pi}^*$ is Π^* with the last row deleted) into Equation 8 yields the short-run portfolio weight equations for the FAIDS model, with short-run adjustment coefficients $\widehat{\Lambda} \widehat{\Pi}^*$. Equation 9 is the Bewley (1979) transformation which allows the long-run coefficients Π^* to be estimated directly.¹² The model is dynamically stable if the eigenvalues of $\widehat{\Lambda}$ have modulus less than unity.

Each equation in (8) contains the lagged portfolio weights of all the other equations (except for the deleted one), allowing for the possibility of the following kind of spillover effect between asset categories: investment in human capital has an indirect effect in raising pension wealth, since it leads to higher future earnings (see, e.g., Johnson, 1996).¹³

⁹ This is the simplest model allowing for time-varying volatilities. More sophisticated models would allow for GARCH effects. However, tests reported below indicate that time-varying volatilities from the simple model did not have a statistically significant impact on determining optimal portfolio weights, so further experimentation using more sophisticated models was abandoned.

¹⁰ A generalization of Christofides (1976), see, e.g., Pesaran *et al.* (1998, p. 52).

¹¹ The partial adjustment model dates back at least to Stone and Rowe (1958). The version used here can be rewritten as a generalized equilibrium correction model with common short-run and long-run adjustment coefficients $\widehat{\Lambda}$: $\Delta \widehat{\theta}_t = \widehat{\Lambda} \Delta \widehat{\theta}_t^* - \widehat{\Lambda} (\widehat{\theta}_{t-1} - \widehat{\theta}_{t-1}^*)$. It is important to reiterate that all the dynamic adjustment in the FAIDS model comes from the slow adjustment of the actual portfolio towards the optimal portfolio in the face of an unchanging investment opportunity set.

¹² It is derived by subtracting $(\mathbf{I} - \widehat{\Lambda}) \widehat{\theta}_t$ from each side of the first row of Equation 7 and rearranging.

¹³ The approach taken here involves short-run dynamic adjustment to the optimal long-run portfolio weights. A different modelling framework has been proposed by Ray (1984). He uses a dynamic PIGLOG cost function which includes lagged portfolio weights and assumes short-term myopic optimization by a representative agent. This model is employed by Weale (1986) for example. The two approaches lead to precisely the same estimation equation. Only the interpretation of the coefficients on the lagged portfolio weights differs. In the model used here these coefficients depend on the dynamic adjustment parameters, while in Ray’s case they measure the degree of habit persistence.

The restrictions implied by demand theory in the FAIDS model can be considered as follows. Adding up requires:

$$\sum_{i=1}^N a_i^* = 1, \quad \sum_{i=1}^N c_{ij}^* = 0, \quad \sum_{i=1}^N b_i^* = 0, \quad \sum_{i=1}^N h_{ij}^* = 0 \quad (12)$$

This can be imposed by dropping one equation and inferring its parameters from Equation 12. Homogeneity requires:

$$\sum_{j=1}^N c_{ij}^* = 0 \quad (13)$$

Symmetry requires:

$$c_{ij}^* = c_{ji}^* \quad (14)$$

The wealth elasticity of demand, $\eta_{iW_t} (= \partial Q_{i,t+1} / \partial W_t) / (W_t / Q_{i,t+1})$, can be found from the relationship $\bar{P}_{i,t+1} Q_{i,t+1} = (1 + \bar{r}_{it}) \theta_{it} W_t$ (where $\bar{P}_{i,t+1}$ and $Q_{i,t+1}$ are respectively the expected price and number of units held of asset i at time $t + 1$):

$$\begin{aligned} \eta_{iW_t} &= \left[\frac{\partial \theta_{it}}{\partial W_t} \frac{(1 + \bar{r}_{it}) W_t}{\bar{P}_{i,t+1}} + \frac{(1 + \bar{r}_{it}) \theta_{it}}{\bar{P}_{i,t+1}} \right] \frac{W_t}{Q_{i,t+1}} \\ &= \frac{\partial \ln \theta_{it}}{\partial \ln W_t} + 1 = \frac{b_i^*}{\theta_{it}} + 1 \end{aligned} \quad (15)$$

The uncompensated interest rate elasticity of demand, $e_{ijt} (= \partial Q_{i,t+1} / \partial (1 + \bar{r}_{jt})) / ((1 + \bar{r}_{jt}) / Q_{i,t+1})$, is given by:

$$\begin{aligned} e_{ijt} &= \left[\frac{\partial \theta_{it}}{\partial (1 + \bar{r}_{jt})} \frac{(1 + \bar{r}_{it}) W_t}{\bar{P}_{i,t+1}} + \frac{\partial (1 + \bar{r}_{it}) \theta_{it} W_t}{\partial (1 + \bar{r}_{jt}) \bar{P}_{i,t+1}} \right] \frac{(1 + \bar{r}_{jt})}{Q_{i,t+1}} \\ &= \frac{\partial \theta_{it}}{\partial \ln (1 + \bar{r}_{jt}) \theta_{it}} + \delta_{ij} = \frac{c_{ij}^*}{\theta_{it}} + \delta_{ij} \end{aligned} \quad (16)$$

where δ_{ij} is the Kronecker delta. It is assumed in Equation 16 that (a) because W_t is beginning of period wealth, there is no interest rate effect on wealth, and (b) the expected real yields on different assets are independent of each other. The corresponding compensated elasticity is given by:

$$e_{ijt}^* = e_{ijt} + \eta_{iW_t} \theta_{jt} \quad (17)$$

The elasticities with respect to the additional regressors Z_{jt} are given by:

$$\begin{aligned} \xi_{ijt} &= \frac{h_{ij}^*}{\theta_{it}} z_{jt} \quad (\text{if } Z_{jt} \text{ is in levels}) \\ &= \frac{h_{ij}^*}{\theta_{it}} \quad (\text{if } Z_{jt} \text{ is in the form } \ln Z_{jt}) \end{aligned} \quad (18)$$

III. ESTIMATES OF THE MODEL

The estimated equations are based on the following versions of Equations 8 and 9:

$$\begin{aligned} \theta_{it} &= a_i + \sum_{j=1}^{N-1} \lambda_j \theta_{j,t-1} + \sum_{s=0}^K b_{is} \ln(W_{t-s}(1 + r_{W_{t-s}})) \\ &\quad + \sum_{s=0}^K \sum_{j=1}^{N-1} c_{ijs} \ln(1 + r_{j,t-s}) + \sum_{s=0}^K \sum_{j=1}^M h_{ijs} Z_{j,t-s} + u_{it} \end{aligned} \quad (19)$$

and

$$\begin{aligned} \theta_{it} &= a_i^* + \sum_{j=1}^{N-1} \lambda_j^* \Delta \theta_{j,t-1} + b_i^* \ln(W_t(1 + r_{W_t})) \\ &\quad + \sum_{j=1}^{N-1} c_{ij}^* \ln(1 + r_{jt}) + \sum_{j=1}^M h_{ij}^* Z_{jt} + \sum_{s=1}^K b_{is}^* \Delta_s \ln(W_t(1 + r_{W_t})) \\ &\quad + \sum_{s=1}^K \sum_{j=1}^{N-1} c_{ijs}^* \Delta_s \ln(1 + r_{jt}) + \sum_{s=1}^K \sum_{j=1}^M h_{ijs}^* \Delta_s Z_{jt} + u_{it}^* \end{aligned} \quad (20)$$

where Δ_s is the difference operator $(1 - L^s)$. It is assumed that the representative agent forms expectations rationally, so that expected returns are replaced by contemporaneous returns and the orthogonal expectation errors are subsumed in the equation residuals. To account for adding up, the equation relating to human capital is dropped. In addition, since human capital was estimated under the assumption of a fixed internal rate of return of 3%, there are only $N - 1$ time-varying returns (and standard deviations) in Equations 19 and 20: the intercepts are augmented by $c_{iN} \ln(1.03)$.

Equations 19 and 20 are linear VARs and, in principle, can be estimated using the ‘long-run structural modelling’ approach outlined in Pesaran and Shin (2002, section 5). This approach differs from the conventional ‘empirical identification’ approach of, e.g., Johansen (1995), by recognizing that the long-run restrictions implied by the underlying economic theory are a useful aid to identification. For example, in the case of Equation 20, economic theory provides the following insights: the current values of the portfolio weights, asset returns and total wealth are jointly endogenous, whereas the Z_{jt} are (weakly) exogenous; there is no contemporaneous simultaneity between the portfolio weights; and there are long-run restrictions on the parameters implied by homogeneity (Equation 13) and symmetry (Equation 14). Pesaran and Shin show that the original Cowles Commission approach to identification and estimation can be rescued from the current domination of the purely statistical approach of Johansen and others. Their estimation procedure can be implemented using *Microfit 4.0* (Pesaran and Pesaran, 1997), but is practicable only if the number of parameters to be estimated is small. Unfortunately, Equations 19 and 20 could not be estimated in *Microfit 4.0* for two reasons: there were too

many parameters and the systems turned out not to be VARs, since different sets of variables were statistically significant in different equations. The procedure adopted here maintains the spirit of the Pesaran and Shin approach, but instead uses three-stage least squares to estimate both Equations 19 and 20.^{14, 15, 16}

Table 1 presents the statistically significant impact elasticities and the diagnostic test statistics for the short run model Equation 19, based on four portfolio weights and their corresponding real rates of return (net financial wealth (θ_F, r_F) , housing (and durable assets) wealth (θ_H, r_H) , state pension wealth (θ_S, r_S) , and private pension wealth (θ_P, r_P)).¹⁷ The coefficients of the residual equation for human capital (θ_L) were calculated to ensure adding up (Equation 12). While the final set of equations was estimated using three-stage least squares, the specification for each equation was derived on the basis of single equation instrumental variables. The rule was adopted of including only those explanatory variables for which $t > 2$ (unless this led to one or more of the diagnostic test statistics moving into the rejection region), though in the final specification, the t -ratios are usually significant at higher than the 5% level. This general-to-specific methodology led to each equation having a different set of significant explanatory variables: a Wald test ($\chi^2(35, 49.8) = 32$) indicated the rejection of the hypothesis that there is a common set of explanatory variables, and by implication the VAR version of Equation 19. A Wald test for the exclusion of four time-varying standard deviations of asset returns from each of the four portfolio weight equations produced a test statistic

of $\chi^2(16, 26.3) = 22.8$, indicating that the hypothesis that asset return volatilities were not important for determining the composition of personal sector wealth in the UK over the sample period.¹⁸ However, a Wald test for the exclusion of all the exogenous variables (Z_{jt}) indicates rejection ($\chi^2(68, 88.2) = 2825$), thereby rejecting the simple version of the FAIDS model (Equation 3) in favour of the more general model (Equation 19), but with the exclusion of time-varying asset return volatilities. Apart from this, the diagnostic test statistics indicate that the residuals from the four estimated equations are serially uncorrelated, normally distributed, stationary and homoscedastic.

The key results from this table are as follows. Housing (and durable assets) and state pension wealth have unit impact elasticities with respect to wealth, whereas net financial wealth and private pension wealth are wealth-normal, while human capital is a wealth-luxury. Own-rate elasticities equal or exceed unity, and approach two in the case of housing (and durable assets) and private pension assets. Turning to the cross-rate elasticities, it is noted that positive elasticities denote complements and negative elasticities denote substitutes. The only clearcut result is that net financial wealth and state pension wealth are short-run complements. There are sign differences between all the other asset pairs and it should not be any surprise to find (as seen below) that short-run symmetry is rejected. Five of the exogenous variables discussed in Section II above have no impact effect at all on asset allocation: income (Y), financial deregulation ($FINDREG$), the labour force

¹⁴ Note that these equations contain the $(N - 1)$ cointegrating vectors that are of economic interest, namely the $(N - 1)$ optimal portfolio weight equations, see $\theta_i^* = \Lambda^* x_i$ in Equations 8 or 9.

¹⁵ The instrument set contained the current and lagged values of the weakly exogenous regressors and the lagged values of the endogenous regressors.

¹⁶ Three-stage least squares is also the appropriate estimation technique when the representative agent has rational expectations over the contemporaneous returns on assets in Equations 19 and 20: the unobserved expected returns are replaced by the actual returns and instrumented and the white noise expectation errors are relegated to the equation error (Blake, 1991).

¹⁷ The original intention of this paper had been to model the four pension schemes operating in the UK separately. Blake and Orszag (1999) present estimates of the values of the accrued rights in the two state schemes (the basic state pension and the state earnings-related pension (SERPS) schemes) and the two classes of private schemes (occupational and personal pension schemes). SERPS began in 1978 and personal pension schemes began in 1988, although they had a precursor in the form of retirement annuity contracts issued by insurance companies for the self-employed from 1956. Attempts to model the factors explaining the portfolio weights of the four schemes separately were unsuccessful. To avoid discarding observations, the assumption was made that the two assets that were introduced part of the way through the sample period were in fact available for the whole period but subject to a form of truncation during the years prior to their introduction known as the sample selection problem. This problem was first addressed by Heckman (1979) who designed a two-stage estimation process that yields consistent estimates of the equation parameters. At the first stage, a probit model of the portfolio share equation is estimated by maximum likelihood, with the dependent variable set to zero for years prior to the introduction of the asset, and unity otherwise. Using these estimates, the hazard rate is constructed. At the second stage, the hazard rate is included as a separate regressor in the equation explaining the portfolio weight. The effect of this is to normalize the mean of the equation residuals at zero, so that the equation estimates will also be consistent, although the standard errors will be biased. However, when the system of equations was estimated by three stage least squares using *Limdep* (Greene, 1995), there were signs of multicollinearity and standard errors were not produced. The problem of multicollinearity could only be removed by aggregating the following pairs of variables: the basic state pension and SERPS assets, occupational and personal pensions, and housing and durable assets.

¹⁸ In other words, the allocation of wealth across the very lumpy asset classes considered in this study is dominated by first-moment terms.

Table 1. Portfolio composition equations: impact elasticities and diagnostic test statistics (homogeneity and symmetry not imposed)

	Dependent variables:				
	Q_F	Q_H	Q_S	Q_P	Q_L
Wealth elasticities: (<i>t</i> -ratio)	0.3846 (2.54)	1	0.9898 (0.07)	0.3387 (8.15)	1.0929 (73.65)
Uncompensated interest rate elasticities:					
r_F	1.1985 (4.89)	–	0.2218 (8.11)	0.1377 (6.49)	–0.0418 (15.71)
r_H	0.4515 (4.47)	1.8208 (13.49)	–	0.4168 (5.77)	–0.1460 (17.54)
r_S	6.9235 (3.95)	–	1	–12.7658 (6.52)	0.5792 (4.01)
r_P	–2.1495 (4.29)	–0.6193 (2.85)	–	1.9619 (4.69)	0.1234 (3.76)
Other elasticities:					
<i>YVOL</i>	–0.0208 (5.25)	–	–0.0321 (8.95)	–0.0098 (3.00)	0.0047 (0.23)
\dot{P}_F	–	–	0.0437 (5.23)	–0.1238 (5.97)	0.0063 (2.62)
<i>YOUTHDR</i>	–	–	3.1613 (5.63)	–	–0.2590 (107.61)
<i>AGEDR</i>	5.7846 (5.73)	–	–1.9726 (3.86)	–	–0.2071 (67.81)
<i>LIFEXP</i>	–39.9463 (6.69)	–	20.0849 (5.96)	–	0.9005 (203.37)
<i>U</i>	–0.1179 (4.15)	–	0.2389 (9.53)	0.0548 (5.17)	–0.0164 (38.23)
<i>AGEPR</i>	–	–	0.3720 (6.05)	–	–0.0305 (97.82)
<i>GS</i>	–0.0121 (6.13)	0.0032 (3.56)	0.0043 (2.99)	–	0.0001 (0.03)
<i>SURPLUS</i>	0.0139 (4.85)	–	–0.0165 (8.08)	–	0.0005 (0.89)
Diagnostic test statistics	θ_F	θ_H	θ_S	θ_P	θ_L
R^2	0.9975	0.9805	0.9710	0.9976	–
<i>DW</i>	2.32	2.44	2.45	2.27	–
Serial correlation $F(1, 42, 251)$	1.4733	2.4362	2.2684	0.7850	–
Normality $\chi^2(2, 5.99)$	1.2629	1.1642	0.2799	2.5034	–
Unit root (ADF(1)) test $t(-2.93)$	–6.4235	–5.0871	–6.6350	–4.7406	–
ARCH (6) test $\chi^2(6, 12.6)$	5.9466	4.3682	4.0221	2.8248	–

Notes:

1. The estimates are three-stage least squares over the period 1951–1994.
2. The estimates and diagnostic tests are from *EViews 3* (1997).
3. Brackets besides the F and χ^2 tests indicate the degrees of freedom and the corresponding 5% critical value.
4. For the wealth elasticities and the own-interest rate elasticities, the null for the t -test is that these elasticities are unity; in all other cases, the null is that the elasticities are zero.
5. The elasticities are calculated at the point of sample means for the exogenous variables and the end-of-sample observations for the portfolio weights.
6. While the equations are estimated in asset-share (θ_i) form Equation 19, the elasticities are calculated with respect to asset levels (Q_i), see Equations 15–18.

participation rate (*LABPR*), corporate savings (*CS*), and pension fund surpluses (*SURPLUS*).¹⁹

Table 2 presents the results from testing the restrictions of demand theory. The restrictions of homogeneity (Equation 13) and symmetry (Equation 14) separately and together are rejected in the case of the short-run demand system Equation 19. However, when these restrictions are tested in the long-run demand system (Equation 20), they cannot be rejected.

Table 3 presents the long-run elasticities for the FAIDS model Equation 20, with homogeneity and symmetry imposed. Unlike the impact elasticities, the quantity held in the long run of each asset category is, in principle, influenced by the full set of explanatory variables, as a result of each equation's dependency on the lagged portfolio weights of all other assets. It is found that only two of the variables discussed in Section II above, namely the indicator of financial deregulation

¹⁹The other exogenous variables have the following impact effects. Greater income uncertainty (*YVOL*) causes individuals to reduce their weighting in all asset categories, except human capital. The inflation loss on financial assets (\dot{P}_F), while having no immediate effect on financial asset holdings, induces a short-run switch away from personal pension assets towards state pension assets and human capital. An increase in the youth dependency ratio (*YOUTHDR*) induces a switch away from human capital towards state pension wealth, and an increase in the age dependency ratio (*AGEDR*) causes a switch away from state pension wealth and human capital towards net financial assets, while increasing life expectancy (*LIFEXP*) has the opposite impact effect. A rise in unemployment (*UN*) induces a switch away from net financial assets and human capital towards pension assets, while a rise in the age participation rate (*AGEPR*) has the effect of raising the weight in state pension assets at the expense of human capital. Government savings (*GS*) complement housing and state pension assets at the expense of net financial assets, and pension fund surpluses (*SURPLUS*) raise the demand for net financial assets and reduce the demand for state pensions.

Table 2. Tests of the restrictions implied by demand theory

	Wald test statistic	
	Short-run	Long-run
Homogeneity	128.85 ($\chi^2(4, 9.5)$)	0.84 ($\chi^2(4, 9.5)$)
Symmetry	40.82 ($\chi^2(3, 7.81)$)	3.16 ($\chi^2(6, 12.6)$)
Homogeneity and symmetry	359.22 ($\chi^2(7, 14.1)$)	9.61 ($\chi^2(10, 18.3)$)

Note: These test statistics were calculated from three-stage least squares estimates using *EViews 3* (1997).

(*FINDREG*) and the labour force participation rate (*LABPR*), do not have a statistically significant impact in at least one equation of Equation 20. However, a comparison between Tables 1 and 3 shows some striking differences between the impact and long-run elasticities, in terms of both sign and size.

Of particular importance are the long-run wealth elasticities. Net financial wealth is a wealth-inferior asset, with a long-run elasticity of -3 . The pension assets have long-run wealth elasticities that do not differ significantly from unity. Human capital and housing (and durable assets) are wealth-luxuries, with the latter category having a long-run elasticity of three. Turning to the interest rate

Table 3. Portfolio composition equations: long run elasticities (homogeneity and symmetry imposed)

	Dependent variables:				
	Q_F	Q_H	Q_S	Q_P	Q_L
Wealth elasticities: (<i>t</i> -ratios)	-3.1605 (5.93)	3.1055 (6.93)	0.4863 (1.82)	1.1700 (0.78)	1.0784 (32.12)
Uncompensated interest rate elasticities:					
r_F	1.4334 (123.91)	0.2214 (2.17)	-0.7530 (3.69)	0.1429 (1.80)	0
r_H	0.3552 (2.17)	0.7623 (35.49)	-0.4038 (0.94)	0.4347 (1.43)	0
r_S	-0.9679 (3.69)	-0.3236 (0.94)	4.7416 (59.00)	-2.6491 (2.48)	0
r_P	0.1792 (1.80)	0.3398 (1.43)	-2.5847 (2.48)	3.0715 (54.93)	0
Other elasticities:					
Y	4.2152 (5.99)	-0.3121 (1.01)	0.4980 (1.61)	0.2986 (1.23)	-0.3014 (8.95)
$YVOL$	0.0395 (1.70)	0.0100 (0.96)	-0.0423 (3.91)	0.0132 (1.51)	-0.0011 (0.01)
\dot{P}_F	-0.1262 (2.02)	0.0308 (1.13)	0.0266 (0.96)	-0.0028 (0.14)	0.0029 (0.55)
$YOUTHDR$	-2.7600 (3.20)	0.2345 (0.65)	0.6289 (1.76)	-1.2318 (4.42)	0.1989 (104.10)
$AGEDR$	-2.3939 (0.32)	-0.6995 (1.13)	0.1672 (0.27)	-1.1688 (2.51)	0.1224 (34.26)
$LIFEXP$	-25.3563 (1.70)	-29.6547 (4.75)	7.8939 (1.25)	10.8258 (2.25)	3.1360 (349.32)
U	0.0463 (0.69)	0.0101 (0.35)	0.1108 (3.77)	0.1105 (5.01)	-0.0219 (32.50)
$AGEPR$	0.2332 (1.11)	0.2666 (2.93)	0.0342 (0.34)	0.1602 (1.93)	-0.0577 (69.04)
GS	-0.0063 (1.38)	0.0096 (4.57)	-0.0050 (2.11)	0.0039 (2.07)	-0.0005 (0.10)
CS	-0.0630 (0.40)	-0.1514 (2.24)	0.0412 (0.61)	0.0107 (0.20)	0.0153 (1.57)
$SURPLUS$	0.0410 (3.55)	-0.0186 (3.36)	0.0036 (0.68)	0.0011 (0.27)	-0.0011 (0.68)

Notes:

1. See notes to Table 1.

2. The zero uncompensated interest rate elasticities for human capital arise because (a) homogeneity has been imposed and (b) the own rate for human capital is assumed to be constant and so is not included as a separate regressor.

elasticities, it is found that all the own-rate elasticities are positive and, with the exception of housing, exceed unity. The long-run symmetry restrictions indicate that: net financial assets complement all other assets, except state pension assets; housing (and durable assets) complement private pension assets, but are a substitute for state pension assets; state pension assets are long-run substitutes for all other assets, in particular, private pension assets; and private pension assets complement other categories, apart from state pensions.

The exogenous variables have the following long-run effects. While having no short-run effect, income (Y) has a statistically significant long-run effect on net financial assets and human capital. The first asset is a strong income-luxury, while the second is income-inferior: so the income effects for these two asset categories are the precise opposite of the corresponding wealth effects. Greater income uncertainty ($YVOL$) reduces the allocation to state pension wealth. The inflation loss on financial assets (P_F), which is not adequately compensated in their real returns, causes a long-run shift away from financial assets.

Of the demographic variables: an increase in the dependency ratios of both the young and old ($YOUTHDR$ and $AGEDR$) induces greater human capital accumulation at the expense of both net financial and private pension assets, while an increase in life expectancy ($LIFEXP$) causes large switches away from financial and housing assets towards pension assets and human capital accumulation.²⁰ In terms of labour market factors: a rise in unemployment (UN) lowers human capital accumulation in favour of pension asset accumulation, while a rise in the age participation rate ($AGEPR$) reduces the need to accumulate human capital, but raises the demand for housing.²¹

Finally, in Table 4, the speed of adjustment is reported of each portfolio share to a shock to each equation of Equation 19 equal in size to the estimated standard error of that equation, with the adjustment to the human capital share determined residually. The table indicates that the adjustment is fairly slow. It takes an average of 11 years for private pension wealth to fully adjust, and 18 years for both net financial wealth and human capital to fully adjust, with housing and state pension wealth averaging 16 years. Clearly, net financial wealth is used to make key indirect adjustments to the other portfolio weights, otherwise its portfolio adjustment would be much more rapid.

Table 4. *Impulse response analysis*

	Dependent variables:				
	Q_F	Q_H	Q_S	Q_P	Q_L
Number of years for 100% adjustment:					
to shock to Q_F	19	15	12	13	18
to shock to Q_H	16	13	18	14	17
to shock to Q_S	15	18	18	11	14
to shock to Q_P	20	16	15	16	21
Average	18	16	16	11	18

Note:

1. The table reports the time in years for equilibrium to be re-established following an additive shock to each estimated equation in Equation 19 equal to the equation's standard error, respectively, 0.001 359, 0.001 652, 0.001 123, and 0.000 805.

IV. CONCLUSION

There have been substantial changes in the UK personal sector's asset allocation over the post-war period. The share of net financial wealth in total wealth has been on a falling trend, while the shares of housing and pension wealth have been on a rising trend. The share of human capital has been stable within a relatively narrow band, between 75% and 81% of total wealth. It has been examined whether these changes could be explained by a FAIDS model with standard wealth and interest rate effects, but augmented by factors relating to the income, demographic and labour market status, capital market imperfections, and spillover effects from other sectors.

It was found that wealth effects were very important for determining trend shifts in asset allocations, and certainly more important than relative returns. The main explanation for the declining portfolio weight in net financial wealth was the combination of rising per capita wealth over the post-war period and a negative long-run wealth elasticity. In contrast, positive wealth elasticities explained much of the rise in portfolio weights in the other asset categories. It was found that net financial wealth, housing wealth and private pension wealth were complements, and each was a substitute for state pension wealth.

While an index of financial deregulation was not statistically significant, implying that capital market imperfections did not have a pervasive influence on the asset allocation over the sample period, there was some evidence that liquidity constraints were present, but only in respect of financial asset holdings and human capital: the

²⁰ Increasing life expectancy *does not* increase human capital which is measured by the discounted value of *career not lifetime* earnings.

²¹ The remaining elasticities are hard to explain. There is evidence of a long-run spillover effect from the government sector, with government savings (GS) acting as a complement to housing and private pension wealth and a substitute for state pension wealth. Corporate savings (CS) substitute for housing assets, while rising pension fund surpluses ($SURPLUS$), although having no long-run effect on the demand for private pension wealth, induce a small but significant shift between housing and financial assets.

former is subject to a strong positive current income effect, while the latter is subject to a small but significant negative income effect.²² Income volatility had little long-run impact, except to lower state pension wealth. The inflation loss on financial assets reduces the long-run holdings of financial assets without inducing any significant switch towards other assets, and results presented in Blake (2004, Table 3) indicate that the inflation loss on financial assets also lowers long-run consumption: so inflation appears to exert a deadweight loss on the personal sector.

Demographic variables were also found to be important, especially rising longevity which induces substantial switches in the portfolio towards both pension and human capital accumulation and away from financial and housing assets. An individual's status in the labour market also impacts on the asset allocation, with unemployment, for example, causing a switch away from human capital accumulation towards pensions. Finally, it is concluded that the good fit of the estimated equations provides evidence against the hypothesis that the illiquid or mandatory or contractual nature of some asset categories limits the ability of individuals to alter their holdings in other asset categories in order to achieve a desired long-run portfolio across all categories, although the speed of adjustment appears to be very slow.

The analysis offers some important indicators to the future composition of personal sector wealth in the UK. Rising future per capita wealth will sustain the switch from direct holdings of financial assets towards the indirect holdings of financial claims in the form of funded private pension assets and towards housing assets, although this will be attenuated by the high income elasticity on net financial assets.^{23,24} On the other hand, increased longevity will help to switch personal sector wealth away from financial and housing assets towards both human capital and pension asset accumulation. Further, given its status as a substitute, any further reduction in state pension provision will raise the demand for all assets, especially private pension assets. There will therefore continue to be a strong demand for financial assets mainly by pension funds: this will encourage the supply of new financial assets. The demand for housing assets will also continue to grow (although the growth will be attenuated by the consequences of increased longevity). However, a key question is how far the increase in

demand for housing assets will increase the supply as opposed to just the value of the housing stock; but that is partly a public policy question.

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²² The stable long-run share of human capital in total wealth is explained by the combination of a long-run wealth elasticity exceeding unity and a negative long-run income elasticity.

²³ Between 1994 and 2000, the value of private pension assets grew by 12.0% p.a. (Office for National Statistics, 2001a, Table A61), the value of financial assets grew by 10.5% p.a. (Office for National Statistics, 2001a, Table A64), and the value of housing assets (including non-marketable tenancy rights) grew by 9.1% p.a. (Office for National Statistics, 2001b).

²⁴ The model estimated in Table 3 might not be valid in the very long run in the absence of offsetting factors, since it predicts that, if wealth continues to increase, the portfolio weight for housing will rise to unity, while that for net financial wealth will eventually approach zero. However, income appears to provide, at least partially, such an offsetting factor.

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APPENDIX: DATA CONSTRUCTION

Some of the data used in this study were taken from publicly available sources; other data needed to be constructed. Table A1 lists definitions and sources. All the level variables are measured in real per capita terms, while the rates of return are in real terms.

The wealth variables and the rates of return on these needed to be constructed. A total of eight categories of personal wealth were used:²⁵ net financial wealth, housing wealth, consumer durable assets, basic state pension wealth, state earnings related pension scheme (SERPS) wealth, occupational pension wealth, personal pension wealth, and human capital. The construction of the first seven categories was undertaken in Blake and Orszag (1999) and the data for these variables are taken directly from this source. In this section, it is shown how human capital and the rates of return were constructed on the first seven categories of wealth.

Human capital

Human capital is estimated as the expected present value of career earnings for the whole adult population:

$$\begin{aligned}
 WL_t = & \sum_{i=1}^{NM} \sum_{j=0}^{65-A_i} \alpha_i^M \bar{Y}_t^M \left[\frac{(1+g_p)(1+g_y)}{1+k} \right]^j N_{it}^M P_{ijt}^M \\
 & + \sum_{i=1}^{NF} \sum_{j=0}^{60-A_i} \alpha_i^F \bar{Y}_t^F \left[\frac{(1+g_p)(1+g_y)}{1+k} \right]^j N_{it}^F P_{ijt}^F \quad (A 1)
 \end{aligned}$$

²⁵ Although these were aggregated into five categories for the analysis in Section III.

Table A1. Data definitions and sources

Symbol	Name	Definition	Source
W	Wealth ¹	Total personal wealth (£'000 (1990) per capita)	Blake and Orszag (1999)
W_F	Net financial wealth ¹	Gross personal financial assets <i>minus</i> Gross personal financial liabilities (£'000 (1990) per capita)	Blake and Orszag (1999)
W_H	Housing (and durable assets) wealth ¹	Value of housing stock (£'000 (1990) per capita, W_{H_0}) <i>plus</i> Value of personal holdings of durable assets (£'000 (1990) per capita, W_{Dur})	Blake and Orszag (1999)
W_S	State pension wealth ¹	Value of accrued rights in the basic state pension scheme (£'000 (1990) per capita, W_{BSP}) <i>plus</i> Value of accrued rights in the state earnings related pension scheme (£ '000 (1990) per capita, W_{SERPS})	Blake and Orszag (1999)
W_P	Private pension wealth ¹	Value of accrued rights in occupational pension schemes (£'000 (1990) per capita, W_{Occ}) <i>plus</i> Value of assets in personal pension schemes (£'000 (1990) per capita, W_{Per})	Blake and Orszag (1999)
W_L	Human capital ¹	Expected present value of lifetime earnings (£'000 (1990) per capita)	Equation (A 1)
r_F	Return on net financial wealth ³	Weighted rate of return on gross personal financial assets <i>minus</i> Weighted rate of interest on gross personal financial liabilities (% real)	Equation (A 2)
r_H	Return on housing (and durable assets) wealth ³	Weighted capital gains <i>plus</i> rental yield <i>minus</i> depreciation on housing wealth (% real, r_{H_0}) <i>plus</i> Weighted capital gains <i>plus</i> rental yield <i>minus</i> depreciation on durable assets (% real, r_{Dur})	Equation (A 3)
r_S	Return on state pension wealth ³	Weighted internal rate of return on members' contributions into the basic state pension scheme (% real, r_{BSP}) <i>plus</i> Weighted internal rate of return on members' contributions into SERPS (% real, r_{SERPS})	Equation (A 4)
r_P	Return on private pension wealth ³	Weighted internal rate of return on members' private contributions into occupational pension schemes (% real, r_{Occ}) <i>plus</i> Weighted return on assets in personal pension schemes (% real, r_{Per})	Equations (A 4) and (A 5)
r_W	Return on total wealth	Weighted-average return on five categories of wealth	Equations (A 4) and (A 5)
Y	Personal disposable income ^{1,2}	Total personal income before tax [gross income of employees (including employers' National Insurance contributions and employers contributions to private sector pension schemes) <i>plus</i> gross rental income <i>plus</i> net investment income (total interest receipts <i>minus</i> total interest paid)] <i>minus</i> tax payments on income, employee and employer National Insurance contributions, council tax, and other current transfers (£'000 (1990) per capita)	<i>Economic Trends Annual Supplement</i>

(Continued)

Table A1. *Continued*

Symbol	Name	Definition	Source
<i>YVOL</i>	Income volatility	Absolute value of residuals from regression equation for Y (%)	Hendry (1994, Equation 8)
\dot{P}_F	Inflation loss on liquid assets	Product of retail price inflation rate and W_F (£'000 (1990) per capita)	
<i>FINDREG</i>	Index of financial deregulation	Ratio of outstanding consumer credit to GDP between 1975 and 1988 transformed to equal 0 in 1975 and 1 in 1988; 0 for other years	<i>Annual Abstract of Statistics</i>
<i>YOUTHDR</i>	Youth dependency ratio	Population aged below 15 as a percentage of the total population (%)	Government Actuary's Department
<i>AGEDR</i>	Age dependency ratio	Population aged above 60 as a percentage of the total population (%)	Government Actuary's Department
<i>LIFEXP</i>	Life expectancy	Average age at death (years)	<i>English life Tables</i> , Office of Population, Censuses and Surveys
<i>UN</i>	Unemployment rate	Total unemployment as a percentage of the total workforce ⁴ (%)	<i>Economic Trends Annual Supplement</i>
<i>LABPR</i>	Labour force participation rate	Workforce in employment ⁵ as a percentage of the population aged between 15 and 60 (%)	<i>Economic Trends Annual Supplement</i>
<i>AGEPR</i>	Elderly participation rate	Economically active males (≥ 65) plus females (≥ 60) as a percentage of the population of males (≥ 65) plus females (≥ 60) (%)	<i>British Labour Statistics Historical Abstract 1886–1968</i> (Table 109), <i>General Household Survey 1993</i> (Table 5.3), <i>Labour Market Trends</i> , Office for National Statistics (Table 7.3, July 1997)
<i>SC</i>	Company savings ¹	Financial surplus of companies plus Gross domestic fixed capital formation minus Net capital transfers (£'000 (1990) per capita)	<i>Annual Abstract of Statistics</i>
<i>SG</i>	Government savings ¹	Financial surplus of government plus Gross domestic fixed capital formation minus Net capital transfers (£'000 (1990) per capita)	<i>Economic Trends Annual Supplement</i> , British Economy Key Statistics 1900–1970
<i>SURPLUS</i>	Actuarial surplus in occupational pension schemes ¹	Market value of pension assets minus Value of accrued rights in occupational pension (£' 000 (1990) per capita)	

Notes:

1. This variable is measured in real per capita terms. The nominal variable (in £bn, current prices) is divided by $P \times POP$, where P is the index of retail prices (1990 = 1) and POP is the population aged 15 or above (in millions).
2. Personal savings is defined as: personal disposable income minus consumers expenditure on goods and services. This implies that contributions to the state pension schemes, because they are financed on a pay-as-you-go basis, are not part of aggregate personal savings (they are treated as transfers from current workers to current pensioners), but contributions to private funded pension schemes are part of aggregate personal savings.
3. This variable is a real rate of interest expressed in the form $\ln((1+k)/(1+g_p))$ where k is the nominal return and g_p is the inflation rate ($\Delta \ln P$).
4. The total workforce is defined as the sum of the workforce in employment and the unemployed.
5. The workforce in employment is defined as the sum of employees in employment, the self-employed, those in work-related government training programmes and those serving in HM Armed Forces.

where

NM, NF = number of age ranges into which adult male and female population is divided (16–19, 20–24, 25–29, . . . , 55–59 for women plus 60–64 for men); the state retirement ages for males and females are 65 and 60 respectively,
 $\frac{A_i}{\bar{Y}_t^M}, \frac{A_i}{\bar{Y}_t^F}$ = median age in i th age range,
 \bar{Y}_t^M, \bar{Y}_t^F = average annual male and female earnings in year t ,
 α_i^M, α_i^F = proportion of average annual male and female earnings earned in each age range,
 N_{it}^M, N_{it}^F = number of males and females in the i th range in year t ,
 P_{ijt}^M, P_{ijt}^F = probability at time t of a male and female surviving from age A_i to age $A_i + j$,
 g_y = assumed growth rate in labour productivity,
 g_p = assumed inflation rate,
 k = assumed nominal discount rate.

While a wide choice of possibilities for g_y , g_p and k is conceivable, the same assumptions were made as used by the Government Actuary for calculating the value of SERPS benefits, namely, $g_y = 1.5\%$, $g_p = 5\%$, and $k = 8.15\%$ (implying a real discount rate of $r_L = 3\%$); see Table A2.

Rates of return on personal wealth holdings

In this section, rates of return are derived on the seven categories of personal wealth outlined above.²⁶ They are listed in Table A3.

The (nominal) post-tax rate of return on net financial assets is derived as follows:

$$k_{Ft} = \frac{\sum_{i=1}^{NA} \theta_{it}(d_{it} + g_{it})FA_t - \sum_{j=1}^{NL} \varphi_{jt}d_{jt}FL_t}{FA_t - FL_t} \quad (A2)$$

where

NA = number of financial assets,
 θ_{it} = weight of i th financial asset in value of total financial assets,
 d_{it} = post-tax yield on i th financial asset,
 g_{it} = pre-tax capital gain on i th financial asset (since only about 130 000 individuals pay capital gains tax in the UK),
 FA_t = value of total financial assets,
 NL = number of financial liabilities,
 φ_{jt} = weight of j th financial liability in value of total financial liabilities,

d_{jt} = pre-tax yield on j th financial liability (except mortgages for which post-tax yield is used),
 FL_t = value of total financial liabilities.

The (nominal) post-tax rate of return on housing wealth is:

$$k_{Ht} = g_{Ht} + d_{Ht} - \delta_{Ht} \quad (A3)$$

where

g_{Ht} = pre-tax capital gain on houses (since residential houses do not attract capital gains tax, in general),
 d_{Ht} = post-tax rental yield on residential houses,
 δ_{Ht} = rate of depreciation on housing stock.

Data from Savills Residential Research indicated that a good proxy for d_{Ht} would be the post-tax yield on consols. A rate of depreciation of 1% p.a. is assumed: about 0.5% of the housing stock is scrapped each year and another 0.5% is assumed for maintenance costs. If the mortgage borrowing rate is subtracted from Equation A3 then this would equal the negative of the user cost of housing assets (see, e.g., Miles, 1992).

The return on durable assets is calculated using a formula similar to Equation A3. It is assumed that the rate of depreciation on durable assets is 10% per annum and that their rental yield and rate of capital gains is zero.

The expected rate of return from membership of a defined benefit pension scheme has to take into account the fact that while current pension contributions buy current accrued pension rights, the actual pension benefit does not take place until retirement. The means that accrued benefits have to be revalued and survived to retirement age, dynamized from retirement age until death and then discounted back to the date of accrual in order to find the internal rate of return on the scheme.

Net contributions (K_t) into a defined benefit scheme are usually proportional to current income and in the case of occupational schemes attract tax relief:

$$K_t = \gamma_t(1 - \tau_1)Y_t \quad (A4)$$

where

γ_t = member's contribution rate
 Y_t = member's current income
 τ_1 = marginal tax rate in work.

These contributions buy an accrued benefit (B_t) which is generally related to salary at retirement and is taxable.

²⁶ The eighth category, human capital, is constructed to have a constant real rate of return of 3%; see previous subsection.

Table A2. Values of eight categories of personal wealth, 1948–1994, £bn (1990)

Year	W_F	W_{Ho}	W_{Dur}	W_{BSP}	W_{SERPS}	W_{Occ}	W_{Per}	W_L
1948	24.1	10.7	6.7	4.2	–	1.3	–	155.2
1949	24.9	10.4	6.9	8.5	–	1.4	–	163.0
1950	25.8	11.0	7.0	8.6	–	1.6	–	175.4
1951	26.8	11.9	7.2	10.4	–	1.9	–	190.5
1952	27.6	11.7	7.4	11.1	–	2.2	–	203.9
1953	28.5	11.2	7.6	11.3	–	2.4	–	216.2
1954	29.6	11.0	7.9	11.4	–	2.9	–	231.2
1955	30.8	11.7	8.2	14.2	–	3.5	–	249.9
1956	32.0	12.4	8.5	14.4	–	4.0	0.01	266.0
1957	33.4	12.6	8.8	14.6	–	4.4	0.03	280.4
1958	35.6	12.9	9.2	18.4	–	4.9	0.05	287.5
1959	37.7	13.6	9.6	18.7	–	5.3	0.07	304.1
1960	40.0	14.8	10.1	19.0	–	6.0	0.09	326.2
1961	43.0	16.6	10.5	22.1	–	6.9	0.10	345.3
1962	45.3	18.3	11.8	22.4	–	7.6	0.14	363.0
1963	49.2	20.1	11.6	26.6	–	8.7	0.17	384.3
1964	51.4	23.1	14.4	27.0	–	9.6	0.19	415.4
1965	50.7	25.6	14.4	32.4	–	11.0	0.23	448.6
1966	50.3	27.1	14.5	32.7	–	12.0	0.26	468.7
1967	55.9	29.7	16.3	37.3	–	13.3	0.33	494.1
1968	58.1	32.6	16.1	37.6	–	13.9	0.40	530.9
1969	57.7	36.9	18.2	42.3	–	15.3	0.41	573.7
1970	59.0	39.7	19.6	42.7	–	17.0	0.43	653.0
1971	67.7	46.5	22.9	51.9	–	20.0	0.57	729.2
1972	73.8	63.0	24.9	58.8	–	22.9	0.68	894.6
1973	80.9	90.0	29.7	68.0	–	25.9	0.67	1023.3
1974	70.2	100.6	31.8	88.6	–	33.0	0.54	1180.7
1975	80.3	110.8	37.9	111.5	–	42.0	0.96	1558.8
1976	82.8	123.7	44.7	138.1	–	50.0	1.11	1886.0
1977	105.3	137.1	51.4	145.6	–	65.0	1.86	2088.4
1978	111.0	177.1	60.2	164.5	0.0	90.0	2.45	2368.1
1979	123.2	237.4	70.2	189.1	5.6	120.0	3.22	2707.8
1980	148.6	266.6	78.8	228.2	12.3	150.0	4.60	3379.7
1981	157.5	277.1	84.2	263.9	19.5	180.0	5.92	3761.5
1982	193.5	302.8	90.1	288.8	26.8	215.0	8.78	4138.6
1983	216.8	348.8	97.6	320.0	35.0	255.0	11.60	4514.2
1984	252.5	390.6	104.6	346.8	43.8	295.0	15.19	4928.2
1985	271.3	446.0	112.8	365.2	53.5	330.0	19.66	5363.0
1986	326.1	518.4	119.4	391.1	65.7	360.0	25.00	5848.7
1987	347.6	637.4	130.1	421.8	78.8	401.0	30.00	6361.7
1988	357.5	861.5	147.7	440.9	91.7	441.0	35.00	7043.1
1989	418.3	943.4	164.7	468.9	104.3	503.0	50.00	7795.2
1990	394.6	948.3	179.7	505.2	119.4	556.0	60.00	8607.6
1991	459.6	952.4	191.1	563.1	135.0	605.0	75.00	9393.9
1992	536.5	910.7	197.1	659.0	172.9	623.0	95.00	10078.6
1993	702.2	930.4	201.7	685.5	177.8	711.0	130.00	10491.0
1994	705.0	920.5	208.7	703.0	201.5	743.0	140.00	11041.9

Notes:

- Housing (and durable assets) wealth (W_H) is the sum of W_{Ho} and W_{Dur} ; state pension wealth (W_S) is the sum of W_{BSP} and W_{SERPS} ; and private pension wealth (W_P) is the sum of W_{Occ} and W_{Per} .
- For definitions of variables, see Table A1.

Table A3. Rates of return on seven categories of personal wealth 1948–1994 (Nominal, percentage per annum)

Year	k_F	k_{Ho}	k_{Dur}	k_{BSP}^1	k_{BSP}^2	k_{SERPS}^1	k_{SERPS}^2	k_{Occ}^1	k_{Occ}^2	k_{Per}
1948	-0.6	14.8	-10.0	15.3	14.4	-	-	13.7	13.9	-
1949	-2.7	-1.8	-10.0	15.0	14.7	-	-	13.6	14.0	-
1950	4.9	5.6	-10.0	15.2	15.1	-	-	14.0	14.9	-
1951	1.8	9.1	-10.0	20.6	16.6	-	-	18.4	15.6	-
1952	-0.6	-1.1	-10.0	20.6	17.5	-	-	16.5	15.7	-
1953	10.6	-4.6	-10.0	15.1	17.1	-	-	15.1	15.3	-
1954	19.8	-1.6	-10.0	14.2	17.0	-	-	15.5	15.6	-
1955	3.0	6.2	-10.0	16.6	17.3	-	-	17.7	16.4	-
1956	-4.0	6.4	-10.0	16.7	16.6	-	-	16.5	16.0	23.4
1957	-1.2	2.9	-10.0	15.9	15.7	-	-	13.9	15.5	14.6
1958	21.9	2.9	-10.0	15.0	15.6	-	-	12.9	15.1	37.6
1959	23.8	6.5	-10.0	13.1	15.4	-	-	13.7	14.7	25.3
1960	-0.0	10.2	-10.0	13.5	14.8	-	-	15.3	14.3	5.8
1961	-0.2	13.9	-10.0	15.6	14.6	-	-	14.9	14.0	4.4
1962	3.3	12.4	-10.0	16.4	14.7	-	-	12.9	13.8	22.8
1963	9.5	11.4	-10.0	14.2	14.6	-	-	13.3	13.9	15.5
1964	-3.7	16.4	-10.0	15.5	15.0	-	-	16.1	14.3	4.4
1965	6.2	12.6	-10.0	16.9	15.7	-	-	15.4	14.3	13.4
1966	-3.1	8.0	-10.0	16.1	15.8	-	-	15.1	14.4	7.8
1967	19.4	11.7	-10.0	14.6	15.4	-	-	12.4	14.3	21.2
1968	22.2	11.8	-10.0	16.9	16.0	-	-	16.4	14.9	19.5
1969	-7.4	17.0	-10.0	17.4	16.4	-	-	16.1	14.9	2.1
1970	-1.5	11.4	-10.0	18.3	16.6	-	-	19.5	15.6	8.2
1971	27.6	20.6	-10.0	21.1	17.5	-	-	18.9	16.3	34.4
1972	12.9	41.1	-10.0	19.0	18.3	-	-	20.1	17.6	15.4
1973	-20.2	46.1	-10.0	20.7	19.1	-	-	20.2	18.3	-6.5
1974	-28.0	20.7	-10.0	27.0	20.8	-	-	23.8	19.6	-22.8
1975	83.5	17.5	-10.0	34.6	26.1	-	-	31.4	21.5	63.3
1976	-0.5	18.6	-10.0	27.5	26.8	-	-	22.0	22.1	9.5
1977	36.4	18.2	-10.0	26.9	26.2	-	-	16.4	21.5	46.1
1978	5.3	34.3	-10.0	20.0	26.1	18.1	19.9	19.5	21.3	13.4
1979	5.9	39.3	-10.0	24.6	25.8	20.5	19.8	21.6	21.0	13.4
1980	22.1	20.7	-10.0	28.8	24.9	25.1	19.0	26.0	20.1	26.1
1981	6.4	13.1	-10.0	23.3	24.0	18.3	18.7	19.2	19.6	11.8
1982	22.6	15.8	-10.0	20.4	22.8	15.6	18.7	16.4	19.6	33.0
1983	18.7	23.1	-10.0	16.8	22.2	15.4	18.8	15.6	18.8	20.8
1984	19.7	24.9	-10.0	17.2	20.8	13.4	17.3	13.6	17.4	19.4
1985	14.9	20.2	-10.0	18.1	18.9	15.4	15.5	15.6	15.7	17.9
1986	18.4	23.0	-10.0	15.8	17.5	15.0	14.9	15.1	15.0	21.0
1987	4.7	27.5	-10.0	14.8	16.4	15.0	14.7	15.0	14.7	14.8
1988	5.8	41.5	-10.0	17.1	16.5	16.1	15.1	15.7	14.8	15.0
1989	25.5	14.3	-10.0	19.6	17.0	16.5	15.5	16.1	15.2	24.9
1990	-11.8	5.8	-10.0	21.1	17.5	16.8	15.7	16.5	15.4	-1.9
1991	10.2	6.3	-10.0	17.9	17.9	15.4	15.6	15.2	15.5	15.3
1992	16.1	0.2	-10.0	16.1	18.2	13.8	15.3	13.7	15.2	17.5
1993	23.6	6.4	-10.0	14.2	17.6	11.9	14.9	11.6	14.4	24.6
1994	-8.4	7.9	-10.0	14.9	16.7	11.4	14.1	11.2	13.7	-4.1

Notes:

1. Inflation rate forecast equal to the current inflation rate.
2. Inflation rate forecast equal to the average inflation rate over the previous five years.
3. The real return on housing (and durable assets) wealth (r_H) is calculated as the weighted average of k_{Ho} and k_{Dur} , corrected for inflation; that on state pension wealth (r_S) as the weighted average of k_{BSP} and k_{SERPS} , corrected for inflation; and that on private pension wealth (r_P) as the weighted average of k_{Occ} and k_{Per} , corrected for inflation.
4. For definitions of variables, see Table A1.

Define:²⁷

$$\begin{aligned}
 B_t &= \beta_t(1 - \tau_2)Y_t \left(\frac{(1 + g_p)(1 + g_y)}{1 + k} \right)^M \\
 &\times \left[\sum_{s=0}^L \left(\frac{1 + g_p}{1 + k} \right)^s + \lambda \sum_{s=0}^{L'} \left(\frac{1 + g_p}{1 + k} \right)^{s+L+1} \right] \\
 &= \frac{\beta_t(1 - \tau_2)(1 + g_y)^M \left[(1 - v^L)(1 + r)^L + \lambda(1 - v^{L'}) \right]}{\delta(1 + r)^{M+L}} Y_t
 \end{aligned} \tag{A 5}$$

where

- β_t = annual pension accrual rate (e.g. 1/60th),
- τ_2 = marginal tax rate in retirement,
- M = number of years to normal retirement,
- L = expected pension length of member,
- L' = additional expected pension length of surviving spouse,
- λ = pension fraction of surviving spouse (e.g. 0.5),
- $r = (1 + k)/(1 + g_p) - 1$ = real interest rate,
- $v = 1/(1 + r)$,
- $\delta = \ln(1 + r)$ = force of (real) interest.

The expected nominal return from scheme membership is the discount rate k that equates the present value of benefits in Equation A5 with current contributions in Equation A4 (i.e. the internal rate of return).

The internal rates of return can be calculated on the three defined benefit schemes for the average scheme member who in the case of the state scheme is 41 years old with 21 years to retirement ($M=21$) and in the case of an occupational scheme is 43 years old with 19 years to retirement ($M=19$). It is noted that the rate of return will not depend on the member's current income, Y_t . It will also not depend on the tax rate so long as contributions are tax relieved and the pension is taxed at the same rate as contributions are relieved (i.e. $\tau_1 = \tau_2$): a pension scheme gives a tax subsidy during the contribution phase but (with the exception of the tax-free lump sum) takes it back during the benefit phase. However, the state pension schemes do not give tax relief on contributions, yet the pension is taxable, so

there might be a tax effect with these schemes. Also if the marginal tax rate is lower in retirement than in work, this will generate a tax effect, since the pension scheme then permits tax avoidance rather than simply tax deferral. The rate of return will depend positively on the accrual rate, the growth rate in labour productivity, the assumed inflation rate and on longevity; it will be negatively related to the contribution rate.

The pension length differs according to the type of scheme: occupational pension scheme members tend to experience lighter mortality than the population as a whole and so will tend to draw their pensions for longer. The estimated pension lengths in the state schemes were based on commonly-used mortality tables covering the whole population, namely *English Life Tables* No. 11 (1950–1952), No. 12 (1960–1962), No. 13 (1970–1972), and No. 14 (1980–1982), with interpolation for intervening years. The case of a 'hybrid' 41-year-old male with a 36-year-old spouse was taken. For example in 1981, such a man could expect to live until he was 73 years and his wife could expect to live until she was 79. If this man retired at 62 (i.e. half way between the state retirement age for men and women), his pension length would be 11 years and the surviving spouse's pension length would be six years. In contrast, with occupational schemes, the estimated pension lengths were based on the Institute and Faculty of Actuaries' Tables a(55) and a(90) for annuitants, again with interpolation for intervening years; the expected life-time of an annuitant equals the value of a life-time annuity of one unit when the interest rate is zero. These tables indicate that occupational pension scheme members and their spouses live about five years longer than for the population as a whole.

In the case of the basic state pension scheme, the contribution rate (γ_t) is 2% of the lower earnings limit (LEL). There is no tax relief on contributions into the BSP scheme and although the BSP is taxable, its annual value is below the single individual's personal allowance, so tax effects can be ignored here. In terms of Equation A4, $Y_t = LEL_t$ and $\tau_1 = 0$. The average annual accrual rate (β_t) is 2.42% (i.e. the average of the male and female accrual rates of 1/44th and 1/39th respectively).

²⁷ Note that Equation A5 is the certainty equivalent version to the exact formula:

$$B_t = \beta_t(1 - \tau_2)Y_t \left(\frac{(1 + g_p)(1 + g_y)}{1 + k} \right)^M P_{R,t}^M \left[\sum_{s=0}^{\infty} P_{R,s,t}^M \left(\frac{1 + g_p}{1 + k} \right)^s + \lambda \sum_{s=0}^{\infty} (1 - P_{R,s,t}^M) P_{R,s,t}^F \left(\frac{1 + g_p}{1 + k} \right)^s \right]$$

where

- $P_{R,t}^M$ = probability at time t that the member survives until retirement age,
- $P_{R,s,t}^M$ = probability at time t that the member survives for s years following retirement,
- $P_{R,s,t}^F$ = probability at time t that the spouse survives for s years following the member's retirement.

Since it was not possible to get consistent estimates of these survival probabilities for *both* state and occupational pension scheme members for the *whole* sample period, Equation A5 was used.

In the case of SERPS, there is no tax relief on contributions made by the employee, but the SERPS pension is taxable and the average person is likely to pay tax on it once the BSP has been taken into account. The annual pension accrual rate has been falling over time:

1978–1987	1.25%
1988	1.225%
1989	1.205%
1990	1.188%
1991	1.173%
1992	1.161%
1993	1.150%
1994	1.141%

The GAD surveys of occupational pension schemes found the average member’s gross contribution rate into such schemes has been rising over time:

1956	3.34%
1963	3.50%
1967	3.60%
1971	3.70%
1973	4.08%
1975	4.29%
1983	4.64%
1987	4.72%
1991	4.67%

Interpolation was used for the intervening years. The average annual accrual rate is 1.67% (i.e. 1/60th). Most people commute part of their pension into a tax-free lump sum of 1.50 times their final salary and receive a reduced pension based on the 80ths scale (i.e. 1.25%) but this equates to an accrual rate of approximately 1.67%.

The internal rates of return were calculated on the three DB schemes under two different assumptions concerning the expected inflation rate: (a) that the expected inflation rate was equal to the current inflation rate and (b) that the expected inflation rate was equal to the average inflation rate over the previous five years. Table A3 lists the resulting internal rates of return.

Personal pension schemes are defined contribution schemes and the value of the pension depends exclusively on the size of the contributions and the subsequent investment performance. The minimum net contribution rate into an appropriate personal pension scheme is

equal to the contracted-out rebate on National Insurance contributions as follows:

1987	2.5%
1988–1992	2.0%
1993+	1.8%

The Government Actuary’s Department 1991 survey of occupational pension schemes (Table 6.5.) showed that the average member’s contribution rate into a money purchase scheme was higher than the minimum at 4.95%. The (nominal) rate of return on personal pension scheme assets is defined by:

$$k_{Pt} = \frac{K_t(1 - c_t^m) + WP_t(1 - c^f)}{K_t(1 - \tau_1) + WP_t} (1 + k_t) - 1 \quad (A 6)$$

where

- K_t = contribution amount into personal pension scheme,
- WP_t = value of accrued personal pension assets,
- c_t^m = rate of management charges on contributions,
- c^f = rate of fund management charges (we assume 1%),
- k_t = total (nominal) rate of return on financial assets.

The data for Equation A6 are taken from Table 12 of Blake and Orszag (1999). The rates of return are shown in Table A3.

Table A4 shows the means and standard deviations of the variables used in the analysis. It also shows that most of the variables contain unit roots. However, *LIFEXP* and *YVOL* are stationary. The table also shows that based on a simple Dickey–Fuller test, all the asset returns, except for that on state pensions, are stationary. The properties of the asset returns were investigated in more detail on the basis of the following equation (with K up to three):

$$\ln(1 + r_{it}) = a_i + \sum_{s=1}^K b_{is} \ln(1 + r_{i,t-s}) + \sum_{s=1}^K c_{is} T^s + v_{it} \quad (A 7)$$

Table A5 shows that, on the basis of Equation A7, the first and second moments of all asset returns (including those on state pensions) are stationary, since (a) v_{it} is stationary and (b) v_{it} exhibits an ARCH process. These conditions are necessary, although not sufficient, for the FAIDS model used in this paper, which relies on the assumption that all the moments of the asset return distributions are constant.

Table A4. Summary statistics 1949–1994

Variable	Units	Mean	Standard deviation	Dickey–Fuller test ¹
θ_F	%	6.8999	2.9035	–0.1351
θ_H	%	7.6188	1.1592	–2.7241
θ_S	%	5.2170	0.8475	–6.3365*
θ_P	%	2.7572	1.7042	–1.1571
θ_L	%	77.5070	1.4957	–1.7023
W	£'000 (1990) per capita	157.3242	60.3185	–1.2538
W_F	£'000 (1990) per capita	9.2703	1.7325	–0.4166
W_{Ho}	£'000 (1990) per capita	9.3330	5.9254	–2.1535
W_{Dur}	£'000 (1990) per capita	3.1392	0.5285	–3.1187
W_{BSP}	£'000 (1990) per capita	7.8239	3.2551	–4.4984*
W_{SERPS}^2	£'000 (1990) per capita	2.0103	1.1160	–3.4015
W_{Occ}	£'000 (1990) per capita	5.0093	4.3860	–0.6882
$SURPLUS$	£'000 (1990) per capita	0.3287	1.1571	–1.9433
W_{Per}^3	£'000 (1990) per capita	0.2118	0.3750	2.5998
W_L	£'000 (1990) per capita	120.5421	45.6424	–1.5239
r_W	% (real)	3.0760	1.2630	–4.7939*
r_F	% (real)	1.6139	14.7438	–7.3946*
r_H	% (real)	0.2234	8.2815	–3.8632*
r_S	% (real)	10.5440	0.8494	–2.5245
r_P	% (real)	9.0921	2.3832	–4.2811*
Y	£'000 (1990) per capita	5.9902	1.6410	–2.3253
$YVOL$	%	1.4986	1.2818	–5.8358*
PF	£'000 (1990) per capita	0.5492	0.3318	–2.7730
U	%	4.6043	3.6365	–1.8048
$LABPR$	%	78.9414	1.8455	–1.8642
$AGEPR$	%	12.3213	3.1226	–1.3537
$YOUTHDR$	%	21.8872	1.7647	–1.8947
$AGEDR$	%	18.7574	1.9669	2.0919
$LIFEXP$	Years	79.7223	1.0707	–3.6444*
SG	£'000 (1990) per capita	0.0536	0.2492	–1.5070
SC	£'000 (1990) per capita	0.7569	0.2515	–2.9864

Notes:

1. Dickey–Fuller test with trend; critical value -3.51 ; * indicates variable is stationary about trend, all other variables are nonstationary.
2. From 1979.
3. From 1956.

Table A5. Tests of stationarity of the first and second moments of real asset returns, 1948–1994

	$\ln(1+r_F)$	$\ln(1+r_H)$	$\ln(1+r_S)$	$\ln(1+r_P)$	$\ln(1+r_W)$
Unit root (ADF) test ($t = -5.47$)	–6.33	–6.36	–6.49	–6.13	–6.20
ARCH (6) test ($F(6, 31) = 242$)	0.59	0.33	1.62	0.70	1.28

Notes:

1. Estimates and diagnostic tests are from *Microfit 4.0* (Pesaran and Pesaran, 1997, sec. 6.6).
2. 5% critical values are presented besides test statistics.
3. The unit root test in this table differs from that used in Table A4.