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## What Discount Rate Should be used to Valued Defined Benefit Pension Liabilities?

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## Abstract

In defined benefit pension plans, the primary function of the actuarial valuation is to determine appropriate contribution rates from the plan sponsor. For this reason, the valuation discount rate is based on the expected return on the assets of the pension fund and the definition of the actuarial liability can very according to the funding objectives. The valuation of pension liabilities for financial reporting purposes requires a unique definition of the accrued liabilities and a unique discount rate to value these liabilities. Equilibrium pricing theory can be used to derive a formula for the discount rate that should be used to value final salary pension liabilities. Based on UK data, however, the required risk-adjustment appears to be small. Modern pension accounting standards make no allowance for market risk, but use an ad-hoc approach to allow for default risk. Standards such as SFAS 87 and FRS 17 appear to have adopted a hybrid position between fair value accounting and actuarial funding methods.

## 1. The actuarial valuation

### 1.1 The discount rate used for funding purposes

In a defined benefit plan, the contributions that will be required to meet the liabilities of the plan will depend on (a) the amount and incidence of its future benefit payments and (b) future investment returns on the pension fund assets. As both of these variables are uncertain, the contribution rate must be periodically revised to allow for gains or losses arising from differences between the actual and expected experience of the plan.

If the plan sponsor wishes to aim for a stable contribution rate, the funding method applied must allow for the expected return on the plan assets. Thus, higher *expected* returns should lead to lower *current* contributions. For a mature defined benefit plan with a level benefit outgo, the relationship between investment returns and contributions is illustrated by the stationary fund equation, which is:

$$B = F \times \delta_i + C$$

where:

B = level benefit outgo of mature plan

F = value of pension fund

$\delta_i$ = constant force of return earned on pension fund

C = level contribution income.

If the benefit outgo is constant, the contribution income required to maintain a stationary fund will have a negative linear relationship with the force of return. In a mature final salary plan, the benefit outgo is expected to increase in line with salaries rather then being constant. Thornton and Wilson (1992), however, show that the stationary fund equation also applies to final salary plans provided that the variables B, F and C are redefined as fractions of the active member payroll and that $\delta_i$ is redefined as the force of return minus the force of salary growth.

Actuarial funding methods are not based on the cash-flow projection methodology implicit in the stationary fund equation. Instead, they apportion pension costs to periods of active service and amortise the difference between the value of the fund and the value of the accrued liabilities. Khorasanee (2002), however, shows that the two approaches are mathematically equivalent. Consequently, the valuation discount rate used in actuarial funding methods must be based on the expected return on the plan assets. Anderson (1992) states that the valuation interest rate

"is not an estimate of interest rates generally, but is the assumed rate of investment return on the particular fund we are concerned with".

## 1.2 The meaning of the "actuarial liability"

The stationary fund equation indicates that there is an infinite number of funding strategies that will enable a defined-benefit plan to meet its obligations. The larger the accumulated pension fund, the lower the subsequent contribution rate, and *vice versa*. Actuaries have narrowed the range of possible funding strategies by imposing further constraints, the most usual one being to aim for a stable contribution rate over the lifecycle of the plan. It can be shown that the Projected Unit, Entry Age and Aggregate funding methods all satisfy this constraint for particular new entrant patterns (Khorasanee, 2002).

The "actuarial liability" is the fund that would be required to meet the benefit outgo of the plan under the contribution rate specified by the funding method and the assumptions used by the plan actuary. It follows that the actuarial liability is not uniquely defined and varies from one funding method to another – it is a target value for the fund rather than an amount of money owed to anyone. Anderson (1992) explains this peculiar use of the word "liability" as having arisen from life insurance terminology, where:

"...it is the amount of assets to be set aside for whatever the actual claims turn out to be. In the same way, the accrued liability of a pension plan represents a claim on plan assets."

The reason for calculating the actuarial liability is to determine whether the planned pace of funding is ahead or behind schedule. If the actual value of the fund exceeds the actuarial liability the plan has a "surplus", which can be amortized by an increase in the future contribution rate. If the actual value of the fund is less than the actuarial liability the plan has a "deficit", which can be amortized by a reduction in the future contribution rate.

The actuarial liability is thus a figure calculated by the plan actuary as an intermediate step in the determination of a suitable contribution rate. It is a function of the funding method and assumptions used by the actuary. It will not generally be equal to the cost of insuring the benefits payable on termination of the plan.[1]

[1] Unless the funding method is so defined that the contribution rate is exactly equal to the expected change in the value of the termination benefits.

## 1.3 The role of actuarial judgement

The discount rate used for funding purposes requires actuarial judgement. Firstly, the plan actuary must estimate the expected return on the assets held in the pension fund at the valuation date. Secondly, the plan actuary must decide what, if any, margin is required for prudence.

The estimated return on the plan assets is usually determined by extrapolating historic returns on the various asset classes held in the pension fund, e.g. equities, bonds, cash and property. It is customary to compute historic returns net of consumer price inflation and add these real returns to the expected future inflation rate when projecting forward[2]. The reason for doing so is the belief that real returns will be more stable than nominal returns, because investors allow for the expected depreciation in the value of money when pricing securities (Fisher, 1930). The length of the period over which past returns should be averaged is a matter for actuarial judgement. Too short a period may result in an unreliable sample, whereas too long a period may include economic eras too distant to have any relevance to the future. Wilkie (1995) suggests that geometric means are more appropriate than arithmetic means.[3]

The discount rate used for funding purposes future may be lower than the expected return on the plan assets because of the incorporation of a margin for prudence. This may reflect a desire on the part of the plan sponsor either to enhance benefit security by building up a large fund or to reduce the probability of an increase in the future contribution rate. Thornton and Wilson (1992) suggest that a discount rate 0.5% below the expected return on the fund should be defined as "prudent". Haberman *et al* (2003) argue that such a determination can only be made with the aid of stochastic asset/liability projections. It is clear, however, that the definition of a "prudent" discount rate will depend, to some extent, on the judgment of the plan actuary.

# 2. The challenge from financial economics

## 2.1 The principle of no-arbitrage

An arbitrage is defined as simultaneous transactions that produce a risk-free profit without any investment of capital. An investor might seek to construct an arbitrage by simultaneously borrowing and investing funds in carefully chosen financial instruments. The ability to "short-sell" financial instruments is essential to the construction of arbitrages. This means that funds can be borrowed by the notional sale of specified securities as well by the more conventional method of an interest-bearing loan.

In a liquid and active market, the action of traders should ensure that arbitrages are quickly eliminated. This leads to the principal of no-arbitrage, which states that the market will price assets so that arbitrages cannot be constructed. An equivalent statement of the principle of no arbitrage is the "law of one price". This states that two assets producing equivalent cash-flow

[2] See Lee (1986) and Anderson (1992)

[3] If investment returns are lognormally distributed, there is an equal probability that the return over any future period will be above or below the geometric mean.

streams must have the same market price. If this were not so, one could construct an arbitrage by short-selling the more expensive asset and buying the cheaper asset.

The principle of no-arbitrage underpins the most robust asset-pricing models developed in financial economics, such as the Black-Scholes model for pricing derivatives. In applying such methods to defined benefit plans, the absence of a liquid and active market for pension liabilities may be a problem. The law of one price, nevertheless, suggests a method for pricing defined benefit pension liabilities. If we can represent the liabilities of the plan as a cash-flow stream, and find a portfolio of assets producing identical cash flows, the market value of this portfolio is the implied market value of the pension liabilities.

The possibility of finding a "replicating portfolio" for pension liabilities depends on the extent to which the liability cash-flow stream is predictable. If the liabilities are denominated in monetary units, and the plan is large enough for its demographic experience to be relatively predictable, it might be possible to replicate the liabilities quite closely with a portfolio of fixed interest securities. If the liability cash-flow stream is random, however, no-arbitrage pricing is only possible if assets with similar stochastic properties can be found.

The approach used to determine the actuarial liability for funding purposes is inconsistent with no-arbitrage pricing for several reasons. Firstly, the liabilities are not uniquely defined as a particular cash-flow stream, but depend on the chosen funding method. Secondly, the discount rate used to value the liabilities depends on the expected return on the plan assets. This is clearly inconsistent with the principle of no-arbitrage, as two plans with identical pension liabilities could value these liabilities differently because of the assets held in their respective funds. Thirdly, the incorporation of margins for prudence has no relevance to a market-based pricing approach. Exley *et al* (1997) and Gold (2000) provide detailed critiques of actuarial valuation methodologies from a financial economics perspective.

Valuation methods consistent with financial economics require that (a) the discount rate should be derived from the properties of the liabilities rather than those of the assets used to back the liabilities and that (b) the liabilities should be uniquely defined. These requirements are considered further in Sections 2.2 and 2.3 below.

## 2.2 Equilibrium pricing

The principle of no-arbitrage can be used to price a liability when a portfolio of marketable securities can replicate the cash flows required to settle the liability. There is, however, a more general pricing method that can be used when a replicating portfolio does not exist. This is based on the concept of "equilibrium", which describes the condition of the market when all investors have made optimal investment and consumption decisions. Each investor is assumed to have achieved this "optimal" position by maximizing the expected utility of his/her future consumption.

Panjer *et al* (1998) describe the equilibrium pricing method and show that it gives the same results as the no-arbitrage method when a replicating portfolio exists. Equilibrium pricing, however, generally requires additional, less robust assumptions than no-arbitrage pricing. In particular, one must assume that all investors use the same probability model of the world in optimising their decision-making. Elton and Gruber (1995) refer to this assumption as *homogeneity of expectations*.

Following an approach similar to that described by Cochrane (2001), we now derive an expression for the equilibrium price of a single random cash flow with a known probability distribution at some future time T.

Let: $I_t$ = utility at time t of the future consumption of an individual

$C_k$ = consumption of the individual at discrete time intervals k= t, t+1,…, T–1.

$W_T$ = wealth of the individual at time T

$P_T$ = random cash-flow at time T

$u_k(.)$ = utility function for consumption at time k

$U_T(.)$ = utility function for wealth at time T.

$E_t[.]$ = conditional expectation based on information at time t

We assume that $I_t$ has an additive form given by:

$$I_t = \sum_{k=t}^{T-1} u_k(C_k) + U_T(W_T) \tag{1}$$

At time t, we assume that the investor can buy x units of the random cash flow for a price $P_t$ that we are seeking to determine. The expression for the utility of consumption becomes the following function of x:

$$I_t(x) = u_t(C_t - xP_t) + U_T(W_T + xP_T) + \sum_{k=t+1}^{T-1} u_k(C_k)$$

If the consumption amounts, $C_k$, are optimal in the sense that the $I_t$ is maximised for these values of $C_k$, it follows that:

$$\frac{\partial}{\partial x}\left(E_t[I_t(x)]\right)\Big|_{x=0} = 0 \tag{2}$$

Applying the above condition to the formula for $I_t(x)$ gives:

$$-P_t.u_t'(C_t) + E_t\left[P_T.U_T'(W_T)\right] = 0$$

$$\Rightarrow \quad P_t = \frac{E_t\left[P_T.U_T'(W_T)\right]}{u_t'(C_t)} = E_t\left[P_T.\left(\frac{U_T'(W_T)}{u_t'(C_t)}\right)\right] \tag{3}$$

The ratio of marginal utilities in equation (3) is a random variable known as the *stochastic discount factor*. Although the ratio is defined in terms of the marginal utilities of an unknown individual, the stochastic discount factor must be the same for all investors because the random cash flow can only have one price. Although the cash flow is payable at time T, is it useful to re-express the pricing formula given in equation (3) as a relationship between prices at times t and t+1. This is done by first re-writing equation (3) as:

$$P_t.u'_t(C_t) = E_t\left[P_T.U'_T(W_T)\right] \tag{4}$$

The same formula must apply at time t+1, hence:

$$P_{t+1}.u'_{t+1}(C_{t+1}) = E_{t+1}\left[P_T.U'_T(W_T)\right]$$

We now apply the conditional expectation operator, $E_t[.]$, to both sides of the above equation and use the law of iterated expectations to obtain:

$$E_t\left[P_{t+1}.u'_{t+1}(C_{t+1})\right] = E_t E_{t+1}\left[P_T.U'_T(W_T)\right] = E_t\left[P_T.U'_T(W_T)\right] \tag{5}$$

The right-hand-sides of equations (4) and (5) are now equal, which allows us to write:

$$P_t.u'_t(C_t) = E_t\left[u'_{t+1}(C_{t+1})\right]$$

$$\Rightarrow \qquad P_t = E_t\left[P_{t+1}.\left(\frac{u'_{t+1}(C_{t+1})}{u'_t(C_t)}\right)\right]$$

The ratio of marginal utilities in the above expression is the single-period stochastic discount factor, which to simplify our notation we write as:

$$Z_{t+1} = \frac{u'_{t+1}(C_{t+1})}{u'_t(C_t)} \tag{6}$$

Hence the single period pricing formula becomes:

$$P_t = E_t\left[P_{t+1}.Z_{t+1}\right] \tag{7}$$

The Capital Asset Pricing Model

To convert the equilibrium pricing method into a workable tool we need to express the stochastic discount factor in terms of measurable financial variables rather than marginal utilities. The fact that $Z_{t+1}$ is the same for all individuals gives us an economic interpretation of what it is. In future states of the world in which $Z_{t+1}$ is high, the marginal utility of consumption will also be high. This will occur in times of economic hardship, when aggregate consumption is lower than expected. It follows that the present value of cash-flows contingent on such states of the world will be high, as indicated by equation (6). The reverse is true for prosperous states of the world when aggregate consumption is higher than expected.

To derive the Capital Asset Pricing Model (CAPM), we assume that $Z_t$ is a function of the total wealth in the economy. Although this is not easy to measure in itself, a method of calculating the *change* in this wealth will suffice for the purpose of asset pricing. The change in total wealth is normally estimated by measuring the return on a representative portfolio of stockmarket securities, which is referred to as the *market portfolio*.

We now define the following variables:

$M_t$ = value of the market portfolio at time t

$P_t$ = value of an arbitrary asset A at time t

$B_t$ = value of a risk-free bond at time t

$\delta_t^M$ = force of return on the market portfolio between time t and t+1

$\delta_t^A$ = force of return on asset A between time t and t+1

$\mu_R$ = risk-free force of return

For the market portfolio, asset A and the risk-free bond we can write:

$$M_{t+1} = M_t \exp[\delta_t^M]$$

$$P_{t+1} = P_t \exp[\delta_t^A]$$

$$B_{t+1} = B_t \exp[\mu_R]$$

We assume that $\delta_t^M$ and $\delta_t^A$ are both independent and identically distributed normal random variables, so that:

$$\delta_t^M \sim N(\mu_M, \sigma_M^2)$$

$$\delta_t^A \sim N(\mu_A, \sigma_A^2)$$

We allow that $\delta_t^M$ and $\delta_t^A$ may be correlated over the same time period and define:

$$\sigma_{MA} = \text{Covariance}[\delta_t^M, \delta_t^A]$$

Under the CAPM, we assume that the stochastic discount factor is related to the return on the market portfolio by a function of the form:

$$Z_{t+1} = \exp[g + h\delta_t^M]$$

where g and h are constants.

The single-period pricing formula given in equation (7) must be valid for all assets and all portfolios. We now apply it, in turn, to the market portfolio, asset A and the risk-free bond.

$$M_t = E_t[M_{t+1}.Z_{t+1}.]$$

$$\Rightarrow \quad M_t = E_t\left[M_t.\exp[\delta_t^M].\exp[g + h.\delta_t^M]\right]$$

$$\Rightarrow \quad M_t = M_t.e^g.E_t\left[\exp[(h+1).\delta_t^M]\right]$$

which from the properties of the log-normal distribution gives:

$$1 = e^g.\exp[(h+1).\mu_M + \tfrac{1}{2}(h+1)^2\sigma_M^2] \tag{8}$$

Applying the pricing formula to asset A and the risk-free bond gives:

$$1 = e^g.\exp[h.\mu_M + \mu_A + \tfrac{1}{2}(h^2\sigma_M^2 + 2h\sigma_{MA} + \sigma_A^2)] \tag{9}$$

$$1 = e^g \exp[\mu_R + h.\mu_M + \tfrac{1}{2}h^2\sigma_M^2)] \tag{10}$$

Eliminating $e^g$ between equations (8) and (9) gives:

$$\mu_A + \tfrac{1}{2}\sigma_A^2 = \mu_M + \tfrac{1}{2}\sigma_M^2 + h\sigma_M^2 - h\sigma_{MV} \tag{11}$$

Eliminating $e^g$ between equations (8) and (10) gives:

$$h = \frac{-(\mu_M + \tfrac{1}{2}\sigma_M^2 - \mu_R)}{\sigma_M^2} \tag{12}$$

And eliminating h between equations (11) and (12) gives:

$$\mu_A + \tfrac{1}{2}\sigma_A^2 - \mu_R = \frac{\sigma_{MA}}{\sigma_M^2}(\mu_M + \tfrac{1}{2}\sigma_M^2 - \mu_R) \tag{13}$$

Equation (13) is the standard CAPM formula expressed in terms of log-returns. From the properties of the log-normal distribution, $\mu_A + \tfrac{1}{2}\sigma_A^2$ is the expected return on asset A expressed as a force of interest and $\mu_M + \tfrac{1}{2}\sigma_M^2$ is the same for the market portfolio.

Having obtained a formula for $\mu_A$, the price of asset A can be expressed as a present value calculation. From the properties of the log-normal distribution, the conditional distribution of $P_T$ at time t is given by:

$$\ln(P_T) = \ln(P_t) + N\left(\mu_A(T-t), \sigma_A^2(T-t)\right)$$

Taking conditional expectations of both sides of the above equation gives:

$$E_t[\ln(P_T)] = \ln(P_t) + \mu_A(T-t)$$

$$\Rightarrow \quad P_t = \frac{\exp\left[E_t[\ln(P_T)]\right]}{\exp\left[\mu_A(T-t)\right]} \tag{14}$$

The numerator of equation (14) is equivalent to the median value of the conditional distribution for $P_T$, while the denominator is a discount factor for the period $T-t$ using the force of interest $\mu_A$. From the properties of the log-normal distribution, it can be shown that $\mu_A$ is the geometric mean of the return on asset A expressed as a force of interest.

## 2.3 Defining the liabilities of a final salary plan

For the reasons explained in Section 1.2, actuarial funding methods do not give a unique definition of the liabilities of a defined benefit plan. In this section, we examine alternative definitions that arise from fundamental accounting principles and the "spot-market" model of employee remuneration used in corporate finance (e.g. Bulow, 1982). We assess these definitions by examining the evidence cited by McGill *et al* (1996).

Accounting principles

The accounting principle governing what is shown as a liability is the "accruals" concept. It requires that an expense should be charged to profit and loss in the same period that the business has benefited from that expense, irrespective of when the money is actually paid. Expenses that have been incurred without being settled must be shown as a liability in the company balance sheet.

As pension benefits are a form of employee remuneration, their expense should be charged over the service period of each employee. As these expenses are not settled until the employee retires, the benefits earned prior to the balance sheet date are a liability under the accruals concept. So, the accounting liability must be based on the projected benefits earned from past service – those from future service are not liabilities under the accruals concept.

What are the benefits earned from past service? The simplest answer would be the benefits payable should the employees leave service at the balance sheet date. Unfortunately, this appears to be inconsistent with another accounting principle, the "going concern" concept, which requires the accounts to be compiled on the assumption that that the business has "no need to go into liquidation or curtail the level of operations significantly" (Holmes and Sugden, 1999). Benefit calculations based on the assumption that all the employees leave service at the valuation date, or that the plan is terminated at the valuation date, are not consistent with the going concern concept.

A useful demonstration of the going concern concept is the treatment of assets in company balance sheets, which are generally shown at net book value rather than re-sale value. This is because net book value, defined as cost less accumulated depreciation, is deemed to give a better estimate of the value of the asset to the business as a "going concern". The depreciation charge in any period should reflect the estimated loss in value of the asset due to use. As this is inevitably an inexact science, it is customary to use conventional approaches. The most common is the "straight-line" method, in which a level depreciation charge is made over the estimated useful life of the asset. Holmes and Sugden (1999) observe that the straight-line method "is generally used wherever the equal allocation of cost provides a reasonably fair measure of the asset's service".

On applying these ideas to pension benefits, we can see that the proportion of the projected active-member benefits that should be assigned to past service depends on judgements about the how the value of the pension plan to the business changes over its lifecycle. For a pension plan offering salary-linked benefits, a pension charge expected to be a fixed fraction of the employee payroll appears to be sensible method of apportioning costs. It implies that the pension plan is meeting a fixed share of the cost of remunerating the workforce.

Spot-market remuneration model

Notwithstanding the relevance of accounting concepts, the spot-market model of employee remuneration defines the accrued pension benefits as those payable on termination of employment at the balance sheet date.

Under this model, the benefits provided by the pension plan are taken into account in annual salary negotiations. The employer calculates the benefit accrual over any year as the change in the withdrawal benefit over that year. For a final salary plan, therefore, the benefits earned in any year must include the revaluation of accrued benefits with salary escalation over the year. This follows the principles underlying the Current Unit funding method.

It is assumed that salary negotiations are based on the total remuneration, which includes both actual salary and the value of the pension accrual, as defined above. Thus, the greater pension accrual of an employee with long past service should be offset by lower salary increases, to make the total compensation equivalent to that of an identical hypothetical employee with short past service.

The implications of the spot-market model can be demonstrated by the following hypothetical example, comparing the salaries of two employees whose labour is of equal value to their employer.

Employee A

This employee, aged 50, was paid £30,000 last year. He has 20 years past service in the company plan, which provides a pension of one-sixtieth final salary per year of service at age 65. He has just been offered a 5% salary increase.

Employee B

This employee, also aged 50, has just been offered a job on terms that will include membership of the company pension plan. His labour will be of the same value as Employee A.

Comparison

What salary should the company offer Employee B? To simplify matters, we assume that the plan provides only pension benefits and that deferred pensions are based on salary at exit without revaluation prior to retirement.

Total remuneration of Employee A over following year:

$$= \quad £30{,}000 \times 1.05 + \left( \frac{1}{60} \times £30{,}000 \times 1.05 + \frac{20}{60} \times £30{,}000 \times 0.05 \right) \times \frac{l_{65}}{l_{50}} \times v^{15} \times \bar{a}_{65}$$

Using the AM92 ULT life table and an interest rate of 4% per annum gives:

$$= \quad £31{,}500 + £1{,}025 \times 0.9083 \times 0.5553 \times 11.776$$

Total remuneration of Employee A= £37,588

Let S be the initial salary of Employee B. If both employees are to have the same total remuneration we can deduce that:

$$S + \left( \frac{1}{60} \times S \right) \times \frac{l_{65}}{l_{50}} \times v^{15} \times \bar{a}_{65} = £37{,}588$$

$\Rightarrow$ S = £34,202

Thus Employee B should be offered a salary of £34,202, which is 8.6% above the salary of £31,500 paid to Employee A over the same year. It follows that the spot-market model implies that employees doing the same job would negotiate significantly different salaries because of their different rates of pension accrual.

Discussion of alternative definitions

The spot-market model gives a precise definition of the accrued pension benefits. This is an advantage over the accounting principles discussed above, which provide only conceptual guidelines that require interpretation. The mathematical precision of the spot-market model, however, should not disguise the fact that it is based on assumptions about the labour market and wage bargaining that may or may not hold in practice. These assumptions are:

- that salary levels are negotiated individually rather than collectively;

- that pension accrual is taken into account whenever salaries are renegotiated.

The first of these assumptions cannot be true for an employer that offers a limited number of salary grades. In such cases, employees could only negotiate changes to their grade rather than individual salary levels. This would not be flexible enough to allow for the value of the employee's pension accrual, which depends on both age and past service under the spot-market model.

The second assumption is an empirical matter that has been the subject of a number of studies in the US, reviewed by McGill *et al* (1996). They conclude that the evidence does not support the idea of a wage-pension trade-off in annual salary negotiations. Instead, they argue that final salary pension benefits represent a "compensation premium", designed to attract workers who expect to remain in their positions for long durations and reward them for their loyalty if they do so. Thus, the benefits provided by the plan are determined at entry, for the entire period of future service, in units linked to the employee's salary at exit. As they are not subject to re-negotiation, they should be apportioned to annual service periods on the assumption that the employee's future service is in line with reasonable expectations. This interpretation has an attractive symmetry with the treatment of assets in the company balance sheet, where depreciation charges are determined by spreading the cost of the asset over its expected useful life.

The compensation premium model implies that the employee's pension accrual should be based on projected final salary, as in the Projected Unit funding method. From the accounting perspective, the Projected Unit Method results in the equal apportionment of costs, expressed as a fraction of the employee payroll, over the lifecycle of a plan with a stationary population of active members. As mentioned above, this appears to be a reasonable method of cost allocation, analogous, in some ways, to the straight-line method of depreciation. The Projected Unit Method has in fact been adopted by modern pension accounting standards such as SFAS 87, FRS 17 and IRS 19, and we shall henceforth assume that it provides the correct definition of the accrued pension liability.

For advocates spot-market model, there are two important issues that need to be addressed. The first is that if final salary pension benefits really were included in annual salary negotiations, it would be misleading to present these benefits as "final salary". The plan would actually be providing a stream of annually negotiable deferred annuities based on current salary. It is doubtful whether such a plan, if properly understood by the employees, would be as attractive as a genuine final salary plan based on the compensation premium model described above. Secondly, the use of the Current Unit Method for an immature plan would delay the recognition of pension costs relative to the Projected Unit Method. This would violate the overriding account principle of "prudence" unless the increase in expected costs over time could be justified in terms of an increase the economic value of the plan to the employer over its lifecycle.

# 3. Derivation of the valuation discount rate

## 3.1 The link between equity returns and wage inflation

The link between equity returns and wage inflation is of fundamental importance in valuing pension liabilities using the equilibrium pricing method described Section 2.2. Equation (13) gave the CAPM formula linking the expected return on an arbitrary asset "A" to that on the market portfolio:

$$\mu_A + \tfrac{1}{2}\sigma_A^2 - \mu_R = \frac{\sigma_{MA}}{\sigma_M^2}(\mu_M + \tfrac{1}{2}\sigma_M^2 - \mu_R)$$

As explained in Section 2.2, $\mu_A$ gives the discount rate that would be used to evaluate the present value of asset A from the distribution of its payoff at a future time T. The value of this discount rate depends crucially on the "beta" of asset A, given by:

$$\beta_A = \frac{\sigma_{MA}}{\sigma_M^2}$$

Defining $\rho_{MA}$ as the correlation between the return on the asset and the return on market portfolio, the expression for the beta can be written as:

$$\beta_A = \frac{\sigma_A}{\sigma_M} \times \rho_{MA}$$

Thus, the magnitude of the discount rate for any asset will depend on the extent to which its return is correlated to that on the market portfolio. For the purpose of pricing pension liabilities, it is the covariance between equity returns and wage inflation that will determine the appropriate discount rate. Divided opinions exist on this question, both within the actuarial profession and among economists.

Are equity returns and wage inflation positively correlated?

Dealing with the actuaries first, Thornton and Wilson (1992) argue that equity returns and wage inflation are positively correlated because:

(1) Equity dividends should grow in line with corporate profits.

(2) Corporate profits and wages, being the principal components of national income, should grow at broadly the same rate as GDP.

Allowing for fresh subscriptions of equity capital at an annual rate of between 1% and 2% of total equity market capitalisation, they argue that dividend income per share should grow at between 1% and 2% per annum below GDP growth. They observe that this is broadly consistent with UK experience over the period 1940-1991. The formula for the internal rate of return on holding a share in perpetuity is:

Return = Dividend yield + Dividend growth

Thus, if dividend growth is positively correlated with wage growth, the same must be true of the return on the share.

Hemsted (1962), another actuary, employs a subtly different line of reasoning, as follows:

(1) Equity dividends should grow in line with corporate profits.

(2) Corporate profits should grow in line with shareholders’ funds (i.e. the net assets shown in company balance sheets).

(3) Shareholders’ funds increase over time by the amount of retained profits.

(4) Therefore equity dividends should grow at a rate equal to retained profits divided by shareholders’ funds.

In this model, it is the rate of profit retention that drives dividend growth. Whether the rate of profit retention, or indeed the overall rate of profit, is correlated with either GDP or wage growth is unclear. So Hemsted’s theory, although casting some doubts on Thornton and Wilson’s argument, does not provide a direct answer to our question.

Economists have tended to focus on the question of whether equities offer a hedge against price inflation rather than wage inflation. As wage and price inflation exhibit a strong positive correlation, however, this distinction is of no great importance. If equity returns were positively correlated with price inflation over any time horizon, one would expect a similar positive correlation with wage inflation.

Fisher (1930) proposed that the market prices securities so that their expected real returns (i.e. net of price inflation) should remain constant over time. The Fisher hypothesis subsequently became the starting point for later research examining the statistical evidence for a link between equity returns and inflation. Most of this research concluded that there was no evidence of any positive correlation – indeed over short time horizons the correlation appeared to negative (Bodie, 1976). However, nagging doubts remained about the existence of a positive correlation over long time horizons that would be difficult to corroborate statistically because of limited data. And the existence of such a long-term link is the key question for institutional investors with long-term liabilities, such as pension plans.

Using the discounted dividend model favoured by UK pension actuaries, Campbell and Shiller (1988) provide the following interpretation of the data. An unexpected rise in the inflation rate results in higher interest rates. The market then uses a higher discount rate to determine the present value of future equity dividends, which may cause equity prices to fall in the short term. In the longer term, however, the higher inflation rate will produce higher dividend growth and higher capital appreciation, resulting in a positive correlation between equity returns and inflation over long time periods.

Schotman and Schweitzer (2000) attempt to model both the short-term and long-term effects of inflation by constructing a stochastic model for equity returns that includes:

- a positive contribution to returns from *expected* inflation (the "Fisher effect");
- a negative contribution to returns from *unexpected* inflation (the "inflation beta" effect).

Using a first order autoregressive (AR1) model for the rate of inflation, they conclude that the reliability of equities as a long-term inflation hedge depends greatly on the persistence of inflation, as measured by the autocorrelation parameter. Only when the value of this parameter is 0.9 or greater are equities a good hedge against inflation.

There are limits, however, to what can be achieved by fitting statistical models to historic data. For one thing, there are the problems of model and parameter uncertainty (Haberman *et al*, 2003). More significantly, looking for statistical patterns tells us nothing about the real economic factors that cause one macroeconomic variable to be related to another. These factors can only be understood in terms of the behaviour of economic agents: i.e. firms, investors, consumers and workers acting to advance their interests. Commenting on macroeconomic modelling, Hayek (1931) wrote:

> In fact, neither aggregates nor averages do act upon one another, and it will never be possible to establish necessary connections of cause and effect between them as we can between individual phenomena, individual prices, etc. I would even go so far as to assert that, from the very nature of economic theory, averages can never form a link in its reasoning.

Looking at an individual stock, it is widely accepted that the change in earnings per share is the decisive factor affecting the change in its market price (Elton & Gruber, 1995). This accounts for the importance given to earnings forecasts by analysts and investors. As these forecasts create expectations among investors, changes in expected earnings may drive price movements until the actual earnings are declared, when the difference between actual and expected earnings will be incorporated into the price. Thus, the actual earnings stream will influence the return on the stock in the longer term. The "equity market" is simply a portfolio of individual stocks weighted by market capitalisation, so the arguments are broadly the same. In the short-term, the influence of earnings on the level of the market may be obscured by variations in the price/earnings ratio, but in the long-term the decisive factor will be earnings growth. Shiller (1989) uses a moving average of real earnings as a measure of fundamental value for the equity market.

By "earnings" we mean the profit available for distribution to the ordinary shareholders. The mechanism linking profit growth to equity returns is fairly obvious, but is there a similar connection between profits and wages? On first inspection, we might expect a company paying higher wages to make lower profits, and vice versa. If this naïve reasoning were correct, however, we would be hard pressed to explain the growth in living standards that has occurred in industrialised countries, which has gone hand-in-hand with capital accumulation and profit growth.

At the micro level, there are businesses making "above-normal" profits and businesses making "below normal" profits as part of the natural process of competition and adjustment that occurs in a capitalist economy. The more profitable businesses are generally in their favourable position because they are serving their customers better than their competitors or operating within a booming industry. In either event, the natural impulse for the managers of a successful business is to invest and expand, which involves employing more factors of production, one of which is labour. And an increase in the demand of labour should, other things being equal, bid up the wages of workers.

Unless a business employs a substantial proportion of the workers in its industry, its expansion will have only a small effect on wages, no more than we might expect a new lottery winner to have on the price of a luxury good. But a growing economy is characterised by a general growth in profits brought about by the accumulation of capital, i.e. the expanding businesses outnumber the contracting ones. In such circumstances, the actions of individual businesses will bid up the average level of wages, provided that the rate of capital accumulation exceeds the rate of growth in the number of workers. Thus, the factor driving up wages is the increasing scarcity of workers relative to capital, or as von Mises (1949) wrote:

> What raises wage rates is an increase in capital exceeding the increase in population or, in other words, an increase in the per-head quota of capital invested.

## Statistical evidence

Growth in profits is the causal agent influencing both equity returns and wage growth. The timing of the effects brought about by profit growth, however, is potentially very different. As explained above, equity prices are influenced by *expected* profits as well as actual profits. If profits are expected to rise, perhaps because the economy is emerging from a recession, it is likely the equity market will anticipate this change. The situation is very different for the labour market. At the end of a recession we would expect unemployment to be relatively high, so the increasing demand for labour would have a relatively small effect on wages. The most rapid growth in wages in likely to occur after the economic expansion has continued for some time, when unemployment is low and the labour market is tight.[4]

Putting these arguments together, it follows that the correlation between equity returns and wage growth should occur only after a significant time lag. That is, current equity returns should be positively correlated with *future* wage inflation.

Estimated correlation coefficients for real wage growth and real equity returns, lagged by annual periods of up to four years, are shown in the table below. These were derived from UK data over period 1946-2002, taken from Barclays Capital Equity Index, the Retail Prices Index, the National Average Earnings Index and various other UK wage indices prior to 1963.[5] To be consistent with the pricing model in Section 2.2, these growth rates were expressed as log-returns before the correlations were calculated.

| Correlation of real equity return with real wage growth | | | | | |
|---|---|---|---|---|---|
| Time lag (years) | 0 | 1 | 2 | 3 | 4 |
| Correlation | –0.09 | +0.26 | +0.39 | +0.26 | –0.07 |

The standard error in a correlation coefficient estimated from N years data is $1/\sqrt{N}$. The coefficients for time lags of one and three years are on the borderline of significance at the 5% level. The table above provides reasonable evidence that real equity returns are positively correlated with real wage growth after time lags of between one and three years, which is consistent with our understanding of the fundamental economics.

## 3.2 Applying the CAPM

We now apply the CAPM to determine the discount rate that should used to price a cash flow linked to salary at some future time T.

Let: $S_t$ = salary of an active member at time t

$V_t$ = value at time t of a cash flow payable at time T equal to $S_T$

$\delta_t^S$ = random force of salary growth between time t and t+1

$\delta_t^V$ = random force of return on $V_t$ between time t and t+1

We assume that:

$$\delta_t^V \sim N(\mu_V, \sigma_V^2) \text{ i.i.d.}$$

The relationship between $\delta_t^S$ and $\delta_t^V$ is deduced from:

$$\ln(V_T) = \ln(V_t) + \sum_{k=t}^{T-1} \delta_k^V$$

[4] A statistical investigation of changes in US wage rates over the business cycle is provided by Bils (1985).
[5] The data used for pre-1963 earnings growth were taken from Parsons (1989).

$$\ln(S_T) = \ln(S_t) + \sum_{k=t}^{T-1} \delta_k^S$$

By definition $V_T = S_T$, hence:

$$\ln(V_t) + \sum_{k=t}^{T-1} \delta_k^V = \ln(S_t) + \sum_{k=t}^{T-1} \delta_k^S \quad (15)$$

We now assume the Barclays Capital Equity Index is the market portfolio and propose the following model for $\delta_t^S$ based on the evidence presented in the previous section.

$$\delta_t^S = \mu_S + \sum_{j=0}^{D} a_j \left(\delta_{t-j}^M - \mu_M\right) + \varepsilon_t^S \quad (16)$$

where: $\mu_S$ = unconditional expectation of the force of salary growth

$D$ = duration in years over which salary growth and equity returns are linked

$a_j$ = parameter linking salary growth and equity return at a time-lag of j years

$\varepsilon_t^S$ = random noise term for the force of salary growth between t and t+1

Taking the unconditional covariance of each side of equation (16) with $\delta_{t-k}^M$ gives:

$$\mathrm{Cov}\left(\delta_t^S, \delta_{t-k}^M\right) = \sum_{j=0}^{D} a_j \mathrm{Cov}\left(\delta_{t-j}^M, \delta_{t-k}^M\right)$$

Our assumption that $\delta_t^M \sim N\left(\mu_M, \sigma_M^2\right)$ i.i.d. gives:

$$a_k = \frac{\mathrm{Cov}\left(\delta_t^S, \delta_{t-k}^M\right)}{\sigma_M^2} = \left(\frac{\sigma_S}{\sigma_M}\right) c_k \quad (17)$$

where $c_k$ is the correlation between $\delta_t^S$ and $\delta_{t-k}^M$.

Beta of salary-linked cash-flow

Taking the conditional covariance of each side of equation (15) with $\delta_t^M$ gives:

$$\sum_{k=t}^{T-1} \mathrm{Cov}_t\left(\delta_k^V, \delta_t^M\right) = \sum_{k=t}^{T-1} \mathrm{Cov}_t\left(\delta_k^S, \delta_t^M\right)$$

Non-zero terms on the left-hand-side summation only occur when k = t, hence:

$$\mathrm{Cov}\left(\delta_t^V, \delta_t^M\right) = \sigma_{MV} = \sum_{k=t}^{T-1} \mathrm{Cov}_t\left(\delta_k^S, \delta_t^M\right)$$

Using equation (16) to substitute for $\delta_t^S$ in the above equation gives:

$$\sigma_{MV} = \sum_{k=t}^{T-1} \mathrm{Cov}_t\left(\sum_{j=0}^{D} a_j \delta_{k-j}^M, \delta_t^M\right)$$

Non-zero terms from the inner summation only occur when k – j = t, hence:

$$\sigma_{MV} = \sigma_M^2 \sum_{k=t}^{T-1} a_{k-t}$$

If T>D, the above equation can be re-written as:

$$\sigma_{MV} = \sigma_M^2 \sum_{j=0}^{D} a_j$$

And using equation (17), the beta of the salary-linked cash flow can be expressed as:

$$\beta_V = \frac{\sigma_{MV}}{\sigma_M^2} = \left(\frac{\sigma_S}{\sigma_M}\right) \sum_{j=0}^{D} c_j \qquad (18)$$

<u>Variance of return on salary-linked asset</u>

It is necessary to estimate the unconditional variance, $\sigma_V^2$, in order to determine $\mu_V$. Taking the conditional variance of both sides of equation (15) gives:

$$\mathrm{Var}_t\left(\sum_{k=t}^{T-1} \delta_k^V\right) = \mathrm{Var}_t\left(\sum_{k=t}^{T-1} \delta_k^S\right)$$

As $\delta_t^V$ is i.i.d., the left-hand-side simplifies to the sum of the unconditional variances, to give:

$$(T-t)\sigma_V^2 = \mathrm{Var}_t\left(\sum_{k=t}^{T-1} \delta_k^S\right)$$

Using equation (16) to substitute for $\delta_t^S$ gives:

$$(T-t)\sigma_V^2 = \mathrm{Var}_t\left(\sum_{k=t}^{T-1}\left(\sum_{j=0}^{D} a_j(\delta_{k-j}^M - \mu_M) + \varepsilon_k^S\right)\right)$$

As $\delta_t^M$ and $\varepsilon_t^S$ are i.i.d. and uncorrelated to each other, the right-hand-side can be expressed as a linear combination of their unconditional variances. For T>>D the following approximation holds:

$$(T-t)\sigma_V^2 \approx (T-t).\sigma_M^2.\left(\sum_{j=0}^{D} a_j\right)^2 + (T-t).\mathrm{Var}(\varepsilon_t^S)$$

And using equation (17) to substitute for $a_j$ gives:

$$\sigma_V^2 \approx \sigma_S^2.\left(\sum_{j=0}^{D} c_j\right)^2 + \mathrm{Var}(\varepsilon_t^S) \qquad (19)$$

An expression for the variance of the error term is obtained by taking the unconditional variance of both sides of equation (16):

$$\mathrm{Var}(\varepsilon_t^S) = \sigma_S^2 - \sigma_M^2\left(\sum_{j=0}^{D} a_j^2\right)$$

Using equation (17) to substitute for $a_j$ gives:

$$\mathrm{Var}(\varepsilon_t^S) = \sigma_S^2 - \sigma_S^2\sum_{j=0}^{D} c_j^2 \qquad (20)$$

And eliminating the variance of the error term between equations (19) and (20) gives:

$$\sigma_V^2 \approx \sigma_S^2\left[1 + \left(\sum_{j=0}^{D} c_j\right)^2 - \sum_{j=0}^{D} c_j^2\right] \qquad (21)$$

## 3.3 Fitting the model to UK data

As investors are interested in what their future consumption will be in real terms, we assume that the pricing model derived in Section 2.2 is framed in real currency units (i.e. net of price inflation). It follows that our parameter estimates should be derived from real returns. The following parameter estimates were obtained from UK data over the period 1946-2002:

- $\mu_M = 0.044$, $\sigma_M = 0.21$
- $\mu_S = 0.018$, $\sigma_S = 0.025$
- D = 4
- $a_0 = -0.09$, $a_1 = 0.26$, $a_2 = 0.39$, $a_3 = 0.26$, $a_4 = -0.07$

The parameter for the real risk-free return, $\mu_R$, should be the real redemption yield on a long-dated, zero coupon bond giving a payout indexed in line with price inflation. The nearest UK equivalent would be long-dated index-linked gilts, which currently offer a real annual redemption yield of approximately 2%, so we shall assume:

- $\mu_R = 0.020$

Inserting the estimated parameters into equation (18) gives:

$$\beta_V = \frac{0.025}{0.21}\left(-0.09 + 0.026 + 0.039 + 0.026 - 0.07\right) = 0.089$$

From equation (20) we obtain:

$$\sigma_V = 0.025 \times \sqrt{1 + 0.75^2 - 0.09^2 - 0.26^2 - 0.39^2 - 0.26^2 - 0.07^2} = 0.028$$

And from equation (13) we obtain:

$$\mu_V = 0.02 - \tfrac{1}{2} \times 0.028^2 + 0.089 \times (0.044 + \tfrac{1}{2} \times 0.21^2 - 0.02)$$

$$\mu_V = 0.0237$$

Thus, the discount rate that should be used to value a salary-linked cash flow is about 0.4% per annum above the risk-free return. This small difference is because the estimated beta of the salary-linked asset is only 0.089. If we examine the formula for the beta, we see that:

$$\beta_V = \left(\frac{\sigma_S}{\sigma_M}\right) \times \rho_{MV} = \left(\frac{0.025}{0.21}\right) \times 0.75 = 0.089$$

The beta of the salary-linked asset is small, because the standard deviation of real salary growth is estimated to be less than one-eighth the standard deviation of the real return on the market portfolio. This results in a small risk premium, in spite of the relatively high long-term correlation between equity returns and salary growth.

# 4. Accounting requirements

## 4.1 Consistency and comparability

In its summary of SFAS 87, The US Financial Accounting Standards Board (FASB) states that part of the justification for introducing the standard was that:

> Critics of prior accounting requirements, including users of financial statements, became aware that reported pension cost was not comparable from one company to another and was not consistent from period to period for the same company.

In the UK, the Accounting Standards Board (ASB) makes a similar point in its discussion of the reasons for introducing FRS 17, stating that under the previous accounting standard (SSAP24):

> there were too many options available to the preparers of accounts, leading to inconsistency in accounting practice and allowing a great deal of flexibility to adjust results on a short-term basis

Does using the CAPM to value pension liabilities meet the accounting requirements for consistency and comparability? In fact, there are reasons for doubting that is does. To begin with, it involves a number of assumptions that are not believed to be wholly realistic. Two that give particular cause for concern are:

(1) the equilibrium assumption that all investors have optimised their portfolio and consumption decisions;

(2) the assumption that all investors expect the same distribution of returns from every asset ("homogeneous expectations").

Both of the above assumptions appear to be inconsistent with commonplace observations of investor behaviour. It is also worrying that calibrating equilibrium-pricing models with past consumption and investment data suggests that investors exhibit a wholly implausible degree of risk aversion. This is the famous "equity risk-premium puzzle" of modern finance. While various explanations of the puzzle have been mooted (see Cochrane, 2001), the possibility that the underlying model may simply be unrealistic cannot be disregarded.

Even if the model were correct, parameter estimation would be a further problem. As shown in the previous section, the CAPM formula for the risk-adjusted discount rate requires the following parameters to be estimated.

- the expected return on the market portfolio
- the variance of the return on the market portfolio
- the covariances between the return on the market portfolio and salary growth at different time lags.
- the variance of salary growth.

Although all these parameters can be estimated from past data, consistency and comparability would require clear rules for the choice of data series, the sampling period and the revision of previous estimates when more data became available. The revision of parameter estimates may result in spurious variations in pension liabilities over time.

In any case, the results of the previous section indicate that only a small risk-adjustment is required for the discount rate used to value final salary liabilities. This is because the standard deviation in real salary growth is small compared with the standard deviation in the real return on the market portfolio. Concepts Statement 7, issued by the FASB in 2000, gives the following guidance on estimating risk-adjustments to the value of assets and liabilities:

… in many cases a reliable estimate of the market risk premium may not be obtainable or the amount may be small relative to potential measurement error in the estimated cash flows. In such situations, the present value of expected cash flows, discounted at a risk-free rate of interest, may be the best available estimate in the circumstances.

Thus, given the model and parameter uncertainty involved in applying the CAPM, using an unadjusted risk-free discount rate may be the most practical way of satisfying the requirements for consistency and comparability. If so, our model suggests that a real interest yield should be used to discount real projected pension amounts.

## 4.2 Allowing for default risk

The risk-adjusted discount rate estimated in Section 3.3 depends on the extent to which the price of a cash flow linked to projected salary co-varies with the market portfolio. No allowance was made for the possibility that the employer might default on its pension liabilities.

The ASB and FASB agree that default risk should be allowed for in the valuation of pension liabilities. The problem with doing so is that the risk may vary greatly from one plan to another, depending on factors such at:

- the value of the assets in the pension fund
- the value of the assets and liabilities of the employer's business
- the volatility in the value of the employer's business

The method of pricing corporate debt used in financial economics is described in a classic paper written by Merton (1974). Essentially, the possibility of default is treated as a put option held by the shareholders, which can be valued using the Black-Scholes model. This is a more robust model than the CAPM, depending on the no-arbitrage principle rather than equilibrium assumptions, and importantly gives an answer that does not depend on the expected return on the assets of the business. The volatility of the return on the assets, however, is a critical parameter that would need to be estimated and revised over time. Furthermore, the model makes no distinction between the value of the business on liquidation and the value of the business as a going concern, which is a fundamental distinction in accounting practice. A further complication that would arise for pension liabilities is the existence of a pension fund with a different legal status to the assets of the employer.

While paying due recognition to Merton's approach, the FASB recommends that default risk is allowed for by valuing liabilities using a discount rate obtained from current market yields available on corporate bonds.[6] The ASB follows this approach in FAS17. As very little corporate debt has been issued in the form of index-linked bonds, both SFAS 87 and FRS 17 permit the use of fixed-interest yields on high quality corporate debt (defined as AA status). In defending this approach, the ASB writes in Appendix IV of FRS 17:

[6] See June 2001 issue of "Understanding the Issues", http://www.fasb.org

…although index-linked bonds seem to have been a better match for final salary liabilities, they are not a perfect match and an index-linked bond discount rate would ignore some important aspects of a final salary pension liability, for example the uncertainty of the amounts ultimately to be paid out.

Rather than being used as the discount rate, FRS 17 suggests that the real yield on index-linked bonds should be used in estimating future inflation, as follows:

In jurisdictions where there is a liquid market in long-dated inflation-linked bonds, the yields on such bonds relative to those on fixed interest bonds of similar credit standing will give an indication of the expected rate of general inflation.

It follows that FRS 17 requires that final salary pension liabilities are valued using the following financial assumptions:

- Discount rate = AA corporate bond yield
- Salary growth = Risk-free nominal yield – Risk-free real yield + Real salary growth

As it must always be true that:

Discount rate – Salary growth = Real discount rate – Real salary growth

We can deduce that:

Real discount rate = risk-free real yield + AA corporate bond yield – risk-free nominal yield

In effect, it is assumed that index-linked corporate bonds would have the same default premium as fixed interest bonds issued by the same business. This appears to be a pragmatic approach in the absence of a liquid market for index-linked corporate bonds.

The obvious theoretical problem with using the AA corporate bond yield is that no attempt is made to match the discount rate to the specific risk of default for the business under consideration.[7] Clearly, the AA-rating does not apply to all companies. Moreover, the level of default risk is affected by other specific factors, such as the value of the assets in the pension fund. The adoption of a blanket AA rate appears to be a compromise that allows default risk to be valued in a generic way without permitting specific factors to influence the answer. As these specific factors could only be allowed for through complex financial models requiring subjective parameter estimates, the approach adopted by the accounting standards may be reasonable in light of the need to ensure consistency and comparability.

## 4.3. Fair value accounting

In its glossary to Concepts Statement 7, the FASB defines the fair value of an asset or liability as:

The amount at which the asset (or liability) could be bought (or incurred) or sold (or settled) in a current transaction between willing parties, that is, other than in a forced or liquidation sale.

[7] Another problem is the existence of a range of yields for different AA-rated companies.

Concept Statement 7 advocates the use of fair values for assets and liabilities that represent the present value of future cash flows, which clearly includes pension assets and liabilities. The ASB states that in drafting FRS 17 it was influenced by the "trend internationally towards the use of fair values for pension cost accounting".

If a liquid market exists, as is normally the case for pension fund assets, "fair value" is equivalent to "market value". If no liquid market exists, "fair value" would have to be estimated. The valuation of accrued pension liabilities using a discount rate linked to corporate bond yields is an example of an estimated fair value.

Applying fair value accounting to pension costs would require that:

- the fair value of pension liabilities incurred over the accounting period is charged to profit and loss in the same period;

- experience gains and losses, arising from unexpected changes to the fair values of pension assets and liabilities over the accounting period, are recognised in the same period;

Neither SFAS 87 nor FRS 17, however, are fully consistent with fair value accounting.

Under SFAS 87, pension assets and liabilities are measured at fair value[8]. However, the charge to profit and loss consists of:

- the service cost, which is the fair value of the pension liability incurred over the accounting period; plus

- the interest cost, which is the discount rate applied to the fair value of the liabilities; less

- the *expected* return on the fair value of the assets; plus or minus

- a fraction of previous experience gains and losses.[9]

In addition, only experience gains and losses in excess of a "corridor" equal to 10% of the greater of the assets and the liabilities need be recognised. In justifying the partial and delayed recognition of experience gains and losses, the FASB writes:

> The most relevant and reliable information about [a pension] liability or asset is based on the fair value of plan assets and a measure of the present value of the obligation using current, explicit assumptions. The Board concluded, however, that recognition in financial statements of those amounts in their entirety would be too great a change from past practice.

Clearly, this is not a principled objection to fair value accounting. The motivation behind the approach in SFAS 87 is to smooth out the potential volatility in pension costs that might arise from the full recognition of experience gains and losses. In doing so, the pension charges become closer to the pension contributions actually paid by the employer, which are smoothed over time as a result of actuarial funding methodologies.

[8] Significantly, SFAS 87 allows the use of a smoothed market value for the pension fund assets.
[9] Under SFAS 87, gains and losses must normally be spread over the employees' expected future service.

Under FRS 17, pension assets and liabilities are also measured at fair value. As with SFAS 87, the pension charge includes the service cost, the interest cost and the expected return on the pension fund assets. Experience gains and losses, however, are not recognised as part of the pension charge and do not affect the operating profit for the year. Instead, they are brought into the balance sheet through a statement of total recognised gains and losses. In justifying this approach, the ASB writes in Appendix IV of FRS 17:

> The Board regards actuarial gains and losses as similar in nature to revaluation gains and losses on fixed assets. In relation to the assets in the pension plan, they are held with a view to producing a relatively secure long-term return that will assist in financing the pension cost. The length of the term, coupled with the options available to the employer to restrict the liability in extreme circumstances, means that much of the fluctuations in market value does not affect the relatively stable cash flows between the employer and its pension plan. Market fluctuations are incidental to the main purpose of the pension plan just as revaluation gains and losses on a fixed asset are incidental to its main operating role. They are therefore best reported with the statement of total recognised gains and losses.

This statement merits careful examination. The initial argument that actuarial gains and losses are similar to revaluation gains and losses on fixed assets is reasonable. They are both financial adjustments, unrelated to the operation of the business, which would distort the performance of the company if included in the operating profit. They may therefore be recognised in a separate adjustment to the reserves. However, both FRS 17 and SSAP 24 do indeed allow investment gains to be recognised in operating profit through a reduction in the pension charge. This is what the items for the expected return on the pension fund assets and the discount rate applied to the liabilities are doing. The remainder of the above paragraph attempts to justify this anomaly by citing the incidental nature of “market fluctuations” that do not affect “the relatively stable cash flows between the employer and the pension plans”. But these considerations have no relevance to fair value accounting. If market prices are truly the most reliable values, there is no justification for recognising an investment gain derived from a smoothed, long-term estimate of future returns.

The idea that the short-term market fluctuations are of no great importance to a long-term investor is nevertheless a perfectly respectable view among both pension actuaries and investment practitioners. In actuarial valuations, smoothing procedures are customarily used to bring greater stability to the funding of the plan. Anderson (1992) states that using a smoothed value for the pension fund assets:

> …is often desirable so as to iron out purely random fluctuations in market value which would otherwise cause random fluctuation in cost.

The notion that the fundamental value of a stock to long-term investor may differ from its quoted market price is an axiomatic belief of “value investors”. Graham (1973) provides the following classic statement of this investment philosophy:

> Imagine that in some private business you own a small share that cost you $1,000. One of your partners, named Mr Market, is very obliging indeed. Every day he tells you what he thinks your interest is worth and furthermore offers either to buy you out or to sell additional interest on that basis. Sometime his idea of value appears plausible and justified by business developments and prospects as you know them. Often, on the other hand, Mr Market lets his enthusiasm or his fears run away with him, and the value he proposes seems to you little short of silly. If you are a prudent or sensible businessman, will you let Mr Market’s daily communication determine your view of a $1,000 interest in the enterprise? Only in case you agree with him or want to trade with him. You may be happy to sell out to him when he quotes a ridiculously high price, and equally happy to buy from him when his price

is low. But the rest of the time you will be wiser to form your own ideas of the value of your holdings, based on full reports from the company about its operations and financial position.

This mooted distinction between fundamental value and market value is highly controversial among financial economists. It is clearly inconsistent with the Efficient Markets Hypothesis (EMH) proposed by Fama (1965), which has been the subject of many empirical studies, and was supported by much of the early evidence concerning the behaviour of stock prices. More recently, however, a number of financial economists have raised serious doubts about its validity. The contradictory evidence relates to excess volatility (Shiller, 1989) and mean reversion (Poterba and Summer, 1988), which Cochrane (2001) shows to be different aspects of the same phenomenon. Shiller (2000) and Shleifer (2000) provide behavioural explanations of these observations. The striking feature of these critiques of the EMH is their consistency with the beliefs of both pension actuaries and value investors. Pension actuaries believe that better estimates of long-term value are obtained by smoothing equity market prices over time, implying that much of the observed volatility is unrelated to fundamental value. Value investors believe that volatile sentiment creates a tendency for the market to overreact to information.

The uncertainty about the relationship between price and long-term value puts the compilers of pension accounting standards in a difficult position. It is arguable that, whatever the problems with spurious volatility, fair value accounting is the only method of producing consistent and comparable figures. If so, it should be adopted in its entirety. Instead, SFAS 87 and FRS 17 are peculiar hybrids in which:

- the value of the pension fund assets is disclosed at fair value;
- the value of the liability incurred in an accounting period is estimated at fair value;
- experience gains and losses are either partially recognised or subject to arbitrary smoothing procedures.

If, on the other hand, fair value accounting is deemed not to give a “true and fair” picture, the smoothing mechanisms adopted should apply consistently to both the assets and liabilities of the pension plan. For example, it might be desirable to follow the practice of US pension actuaries in using a smoothed market value for the assets in recognising experience gains and losses, as described by Anderson (1992). If so, the discount rate used to value the liabilities should be smoothed over the same time period. Having carried out these smoothing procedures, gains and losses could be fully recognised in the pension costs figures without further complications.

## 5. Conclusions

The discount rate used in actuarial valuations is derived from the expected return on the assets held in the pension fund. This is an entirely reasonable approach for funding the pension plan. If the plan sponsor wishes to aim for a stable contribution rate over time, it is necessary for the actuary to use best estimates of the future cash flows arising from both the assets and liabilities of the plan. The different possible definitions of the actuarial liability of the plan are also perfectly intelligible, provided that the “actuarial liability” is actually understood as a funding target that will vary according to the assumed new entrant experience of the plan. The terms “actuarial valuation”, “actuarial liability” and “actuarial cost” may nevertheless have created a misleading picture of the true objectives of the funding exercise periodically carried out by the plan actuary.

For the purpose of financial reporting, it is necessary to have an objective definition of the liabilities of the pension plan. It is generally accepted that these should be the liabilities incurred over employees’ service prior to the balance sheet date. There is disagreement, however, over whether these liabilities should be based on the benefits payable on immediate withdrawal (based on current salaries) or the benefits payable at the expected date of withdrawal (based on projected salaries). These alternative definitions arise from different models of how pension benefits are incorporated into wage negotiations. The first definition assumes that pension rights are taken into account whenever salaries are renegotiated, whereas the second definition assumes that they are taken into account only when the employee joins the plan. The evidence appears to support the second definition.

Pricing methods developed in financial economics can be used to derive a discount rate for pension liabilities linked to final salary. As there are no assets that can perfectly hedge these liabilities, the equilibrium pricing approach of the Capital Asset Pricing Model is required. Under this model, the risk adjustment to the discount rate depends on the extent to which the pension liability co-varies with the market. Significant correlations were found to exist between real salary growth and lagged real equity returns in the UK. The required adjustment to the risk-free discount rate is small, however, because the variance of real salary growth is much smaller than the variance of real equity returns. Given the uncertainties associated with the equilibrium pricing method, it may be prudent to ignore this risk adjustment for accounting purposes.

Modern pension accounting standards have sought to make pension costs figures more comparable for different companies, while reducing the scope for manipulation. They have allowed the use of discount rates linked to corporate bond yields rather than government-issued index-linked bonds, because the former implicitly allow for the risk of default. The use of an AA corporate bond discount rate for different companies – as permitted by SFAS 87 and FRS 17 – is an *ad hoc* approach that makes no attempt no allow for variations in default risk for different companies Given the complexities and uncertainties in modelling default risk, however, it may be the only pragmatic solution.

Both SFAS 87 and FRS 17 permit the smoothing of gains and losses in the financing element of pension cost. Under FRS 17 the expected investment gain, rather than the actual investment gain, is deducted from pension cost. The same is true of SFAS 87, although the delayed recognition of actual gains and losses is also required. Service cost, however, must be estimated at fair value in both standards. The result is that these accounting standards adopt a hybrid position between fair value accounting and actuarial funding methods. The accounting bodies should either apply the fair value principle in its entirety or accept that there may be a distinction between the fair value of a pension asset or liability and its long-term value to the business. If smoothing procedures are adopted, they should be applied consistently to both assets and liabilities.

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