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# Longevity risk and the Grim Reaper's toxic tail: The survivor fan charts

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## Abstract

This paper uses survivor fan charts to illustrate the prospective density functions of future male survival rates. The fan charts are based on a version of the Cairns–Blake–Dowd model of male mortality that provides a good fit to recent mortality data for England and Wales. They indicate that although none of us can escape the Grim Reaper, survivorship uncertainty is greatest for males aged a little over 90, confirming that there exists a ‘toxic tail’ for those institutions, such as annuity and pension providers, which are obliged to make payments to them for as long as they live. We also find that taking account of uncertainty in the parameters of the underlying mortality model leads to major increases in estimates of the widths of the fan charts.

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*Keywords:* Cairns–Blake–Dowd model; Mortality; Fan charts; Longevity risk; Survival rates; Toxic tail

## 1. Introduction

There is an ongoing debate about whether human longevity will continue to improve in the future as it has done in the recent past. In the ‘optimists’ camp are demographers such as Vaupel et al. (1998), Tuljapurkar et al. (2000), Oeppen and Vaupel (2002), and Tuljapurkar (2005), who argue that there is no natural upper limit to the length of human life. In the ‘pessimists’ camp are demographers, such as Olshansky et al. (1990, 2001, 2005), Mizuno et al. (2004), and Loladze (2002), who argue that future life expectancy might level off or even decline due to lifestyle and environmental factors, such as obesity and the decreased food-derived health benefits associated with higher levels of atmospheric CO<sub>2</sub>. Other demographers, like de Grey (2006), are critical of the extrapolative forecasting approach adopted by the optimists, but still accept the possibility that scientific advances and the socio-political responses to them might lead to substantial increases in life expectancy over the next century.

This controversy has major financial and economic implications. As people live longer, there is an increasing

burden on those committed to caring for them. For example, pension and annuity providers are obliged to make payments to their policyholders for as long as they live. Until relatively recently, financial planners had assumed that aggregate longevity was improving predictably: annuity providers believed they could protect themselves against longevity risk by holding a diversified annuity book and relying on the law of large numbers to ensure that annuitants died off on average when expected. However, this confidence in the predictability of aggregate longevity has been severely shaken in the last few years by the emerging pensions crisis and events such as the failure of Equitable Life, the world's oldest life office, in the UK in 2000.<sup>1</sup>

The view that longevity will continue to increase is supported by the results in Table 1. The Table shows projected likely survival probabilities based on the Cairns–Blake–Dowd (CBD, 2006) stochastic mortality model calibrated to English and Welsh male mortality data between 1982 and 2002. These survival probabilities are estimated using Monte Carlo

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<sup>1</sup> Equitable Life sold deferred annuities with guaranteed mortality rates, but failed to predict the improvements in mortality between the date the annuities were sold and the date they came into effect (see, e.g., Blake (2001, 2002)).

simulation with two alternative specifications, one assuming that the parameters of the model are estimated with certainty, and the other allowing for possible parameter uncertainty.<sup>2</sup> In the case of certain parameters, Table 1 shows that a man aged 65 in 2002 has a 43.6% probability of reaching 85 and a 24.3% probability of reaching 90. In the case of parameter uncertainty, the corresponding probabilities are a little higher, at 44.2% and 25.3%, respectively. The Table also shows some conditional survival probabilities: for example, the probability of an individual who reaches 85 surviving to 90 is 55.8% with no parameter uncertainty and 57.1% with parameter uncertainty. The general picture is therefore one of continuing strong survivorship, with high probabilities of surviving into and beyond the 80s, and with the effect of parameter uncertainty being to increase likely survival probabilities due to the influence of the positive drift term in the equations generating the parameters.<sup>3, 4</sup>

## 2. The uncertainty of future survivorship

These projections of likely future survival probabilities, while interesting, reveal nothing about the uncertainty attached to future survivorship. One way to represent such uncertainty is through fan charts. These are charts of projected probability densities over each year in a specified forecast period and show the likely confidence interval to which a dynamic quantity may belong in a particular future year. Survivor fan charts provide an excellent framework for illustrating not only the most likely future outcomes, but also the degree of quantitative uncertainty surrounding future survival rates. The best-known fan charts are the Bank of England's inflation fan charts, which have been used with considerable success by the Bank in its efforts to promote public debate on UK monetary policy.<sup>5</sup>

Fig. 1 shows survivor fan charts for men who were 55 years old in 2002, calibrated to the CBD model. The fan chart on the left-hand half of the Figure assumes parameters to be known, while that on the right-hand side makes allowance for them to be uncertain. For each fan chart, the highest and lowest bounds show the central 90% prediction interval over the forecast horizon,<sup>6</sup> the next highest and next lowest mark the bounds of the central 80% prediction intervals, and so on, while the innermost bounds show the central 10% prediction interval. The shading also becomes stronger as the prediction intervals narrow. We can therefore interpret the degree of shading as

<sup>2</sup> The model and how parameter uncertainty is taken into account is explained in the Appendix. Cairns (2000) has more on parameter uncertainty in general, and Olivieri (2001) provides a good discussion the impact of parameter uncertainty on mortality-linked products such as life annuities.

<sup>3</sup> See Eq. (A.2) in the Appendix.

<sup>4</sup> Similar findings have been reported by other studies using simulations based on alternative mortality models (see, e.g., Renshaw and Haberman (2008) who use a version of the Lee–Carter model).

<sup>5</sup> The first inflation fan chart was published by the Bank of England in 1996 (Bank of England, 1996), and inflation fan charts have been published in each of the Bank's quarterly *Inflation Reports* ever since. For more on the fan charts and their implications for public policy, see King (2004).

<sup>6</sup> This means that we can be 90% confident that future survivorship on any given date will lie between these bounds, and 5% confident that it will lie on either of the two tail regions.

Table 1  
Projected survival probabilities

Survival to age $x$	Probability assuming:	
	Parameters certain	Parameters uncertain
Survival probabilities for males aged 65 in 2002		
70	90.8	90.8
75	78.4	78.5
80	62.5	62.8
85	43.6	44.2
90	24.3	25.3
95	9.3	10.2
100	1.9	2.4
105	0.2	0.2
110	0.0	0.0
Survival probabilities for males aged 65 in 2002, conditional on reaching 85		
90	55.8	57.1
95	21.4	23.2
100	4.4	5.4
105	0.3	0.5
110	0.0	0.0
115	0.0	0.0
Survival probabilities for males aged 65 in 2002, conditional on reaching 100		
105	7.7	9.6
110	0.1	0.3
115	0.0	0.0

Notes: Projected survival probabilities reported to 1 decimal point, and calculated as the median projection using 10 000 Monte Carlo simulation trials with the mortality model of Cairns et al. (2006) calibrated on GAD data over the period 1982–2002. The estimated parameters are:  $\hat{\mu} = [-0.0668961480 \ 0.0005904540]^T$ ,  $\hat{\nu} = [0.006114509 - 0.0000939164; -0.0000939164 \ 0.00000150933]$ ,  $A(0) = [-10.95043 \ 0.10582754]^T$  for the certain parameter case, and  $\hat{\mu} = [-0.0668961480 \ 0.0005904540]^T$ ,  $\hat{\nu} = [0.006114509 - 0.0000939164; -0.0000939164 \ 0.00000150933]$ ,  $A(0) = [-10.95043 \ 0.10582754]^T$  for the uncertain parameter case.

reflecting the likelihood of the outcome — the darker the shading, the more likely the outcome.<sup>7</sup>

Both fan charts show that longevity risk – the uncertainty of future survivorship – is very low for the first 20 years, but increases markedly after age 75 and reaches a maximum for ages in the early 90s. Thereafter, it falls off as the cohort becomes extremely old and the survival rate approaches zero. The chart also shows that allowing for parameter uncertainty has a relatively small (positive) effect on the central projection of the future survival rate, but has a very pronounced effect on the width of the fan chart bounds.<sup>8</sup> For example, at age 90, the 90% prediction interval lies between a little under 0.2 and about 0.35 if we ignore parameter uncertainty and is about [0.15, 0.45] if we allow for it. The proportion of men who will survive to their late 80s and early 90s is therefore highly uncertain. For annuity providers, this represents a ‘toxic tail’

<sup>7</sup> Details of the calculations underlying the mortality fan charts are given in the Appendix.

<sup>8</sup> The principal reason for this increased width is uncertainty in the underlying trend rather than in the volatility of mortality rates. As our time horizon increases, uncertainty in the trend dominates all other sources of risk in influencing the width of the fan chart in panel (b).

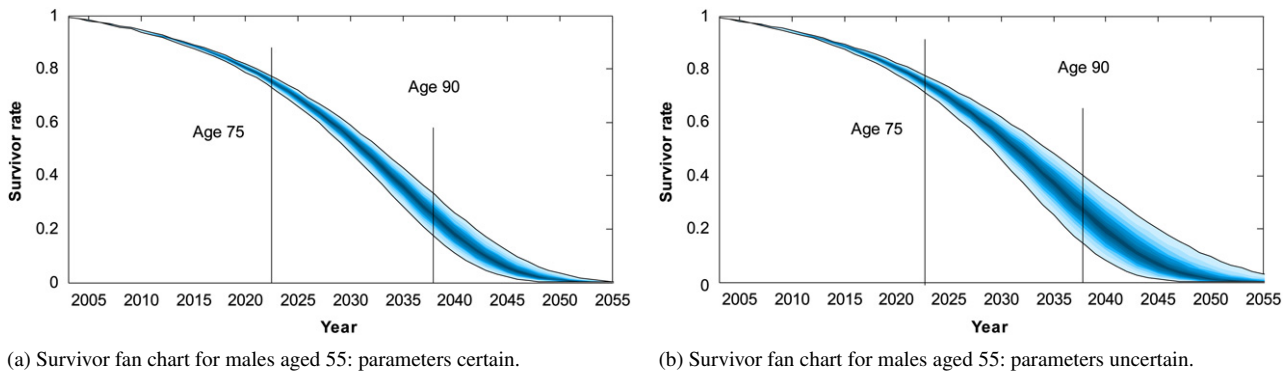


Fig. 1. Survivor fan charts for 55-year old males. Notes: Each chart shows the central 10% prediction interval with the heaviest shading, surrounded by the 20%, 30%, ..., 90% prediction intervals with progressively lighter shading. The bounds of the 90% prediction interval are indicated by black lines for greater visibility. Estimated using 10 000 Monte Carlo simulation trials with the mortality model of Cairns et al. (2006) calibrated on GAD data over the period 1982–2002. The estimated parameters are:  $\hat{\mu} = [-0.0668961480 \ 0.0005904540]^T$ ,  $\hat{V} = [0.006114509 \ -0.0000939164; \ -0.0000939164 \ 0.00000150933]$ ,  $A(0) = [-10.95043 \ 0.10582754]^T$  for the certain parameter case, represented by the left-hand fan chart, and  $\hat{\mu} = [-0.0668961480 \ 0.0005904540]^T$ ,  $\hat{V} = [0.006114509 \ -0.0000939164; \ -0.0000939164 \ 0.00000150933]$ ,  $A(0) = [-10.95043 \ 0.10582754]^T$  for the uncertain parameter case, represented by the right-hand fan chart. The plotted black lines are the bounds of the central 90% prediction interval.

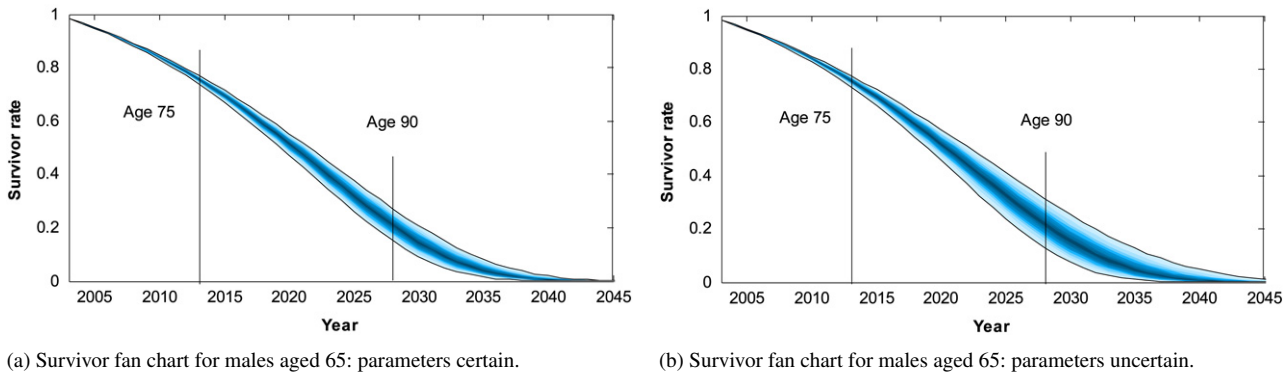


Fig. 2. Survivor fan charts for 65-year old males. Notes: As per Fig. 1.

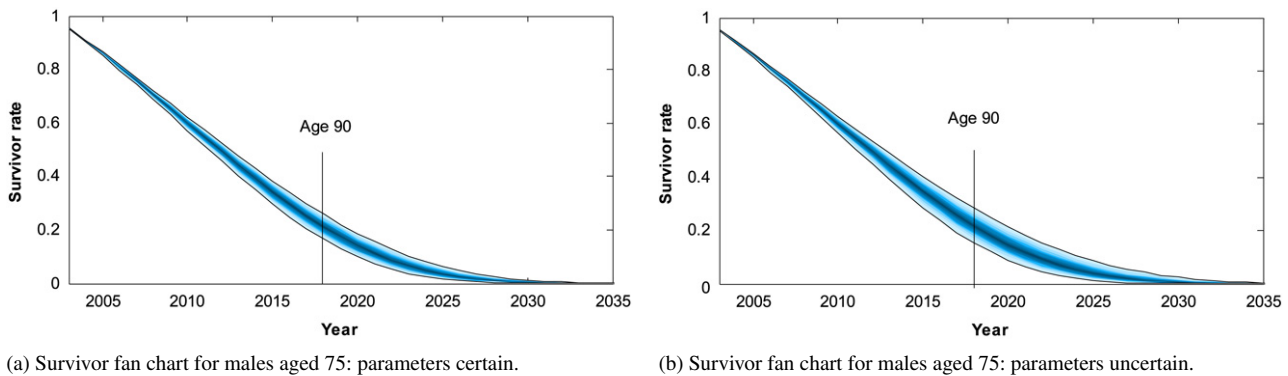


Fig. 3. Survivor fan charts for 75-year old males. Notes: As per Fig. 1.

that can have lethal implications for the annuity provider's financial health.<sup>9</sup> Figs. 2 and 3 give the comparable fan charts for the cohorts of men aged 65 and 75 respectively in 2002.

<sup>9</sup> The credit for inventing this colourful phrase goes to Tom Boardman of the (UK) Prudential. He used it in his April 2006 address to Longevity Two: The Second International Conference on Longevity Risk and Capital Market Solutions Conference in Chicago, but did not include it in his paper published in the conference proceedings (Boardman, 2006).

These show the same features as the earlier fan charts, but also reveal that future survival rates for older cohorts are more uncertain than those for younger cohorts, other things being equal. They also confirm that allowing for parameter uncertainty has a very major impact in widening the dispersion of estimated fan charts. As a rough rule of thumb, allowing for uncertain parameters nearly doubles the dispersion of each of our fan charts over the age ranges where there is serious uncertainty about future survival rates. Failing to allow for

parameter uncertainty, therefore, leads to fan chart forecasts that are far too narrow and grossly underestimate longevity risk.<sup>10</sup>

### 3. Economic and financial implications

To further illustrate the economic and financial implications of this analysis, Table 2 shows the fair prices of current and future annuities under the (more realistic) parameter uncertainty version of the CBD model.<sup>11</sup> The price of a life annuity at different future dates is a good index of the cost of increased longevity. The Table presents annuity price results for males aged 55, 65 and 75 at future horizons of 5, 10, 15 and 20 years ahead. The upper part of the Table gives results assuming interest rates equal to 4% both now and in the future, and these results are very clear: expected annuity prices are set to rise in response to increased longevity, and these rises are positively related to both the age of the men and the length of the horizon. The expected increases vary from 3.8% (for age 55 and  $T = 5$ ) to 18.0% (for age 75 and  $T = 20$ ). Thus, the future cost of providing for increased longevity is predicted to rise considerably.

The Table also presents the bounds of the 90% prediction intervals for the rates of change of future annuity price relative to their current values. These bounds are wide and again increase with both age and horizon. They are also particularly wide for 75 year olds which highlights the ‘toxic tail’ effect. For instance, at a horizon of 20 years, the rate of change of annuity prices for 75-year-old men has a 90% prediction interval equal to  $[-0.2\%, 32.7\%]$ . The cost of providing for future longevity is also clearly very uncertain.

To add another nail to the coffin, the bottom half of the Table shows the comparable results if interest rates should fall to 3% at the end of the horizon periods. These show that a fall in interest rates is likely to produce additional major increases in annuity prices. Such a fall would increase future annuity prices by at least 10% and typically considerably more, for any given longevity scenario. This reminds us that annuity prices are very interest-rate sensitive, and tells us that a conjunction of improved future longevity and lower future interest rates is likely to be especially costly.

### 4. Conclusions

We have shown that fan charts provide a very useful and intuitive means of representing quantitatively measurable uncertainty (or ‘risk’). As such, they also have many obvious related uses. For example, they can be used to obtain estimates

of VaR or expected shortfall risk measures; they can be used for reserving and setting capital requirements; and they can be used for pricing, hedging and general risk management purposes.

This paper has focused on survivor fan charts, and the message they give is very simple. The healthcare system, pension funds, life companies and, indeed, the state itself, are all heavily exposed to longevity risk. Their exposure to this risk – especially their exposure to the ‘toxic tail’ of those who survive into their 90s – therefore needs to be managed, and those institutions that fail to heed this warning are likely to face a very uncertain future themselves, especially if interest rates fall further. However, the news is not all bad. Annuity and pension providers can at least take comfort from one fact: even though longevity is improving, it still looks as though no-one in the foreseeable future will live to the ripe old age of Methuselah.<sup>12</sup>

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### Appendix. The mortality model used to generate the fan charts

The survivor fan charts discussed in this paper are based on an underlying stochastic mortality model set out by Cairns et al. (2006). Let  $q(t, x)$  be the realized mortality rate in year  $t + 1$  (that is, from time  $t$  to time  $t + 1$ ) of a cohort aged  $x$  at time 0. We assume that  $q(t, x)$  is governed by the following two-factor Perks stochastic process:

$$q(t, x) = \frac{\exp [A_1(t + 1) + A_2(t + 1)(t + x)]}{1 + \exp [A_1(t + 1) + A_2(t + 1)(t + x)]} \quad (\text{A.1})$$

where  $A_1(t + 1)$  and  $A_2(t + 1)$  are themselves stochastic processes that are measurable at time  $t + 1$  (see Perks (1932) and Benjamin and Pollard (1993)). Now let  $A(t) = (A_1(t), A_2(t))'$  and assume that  $A(t)$  is a random walk with drift:

$$A(t + 1) = A(t) + \mu + CZ(t + 1) \quad (\text{A.2})$$

where  $\mu$  is a constant  $2 \times 1$  vector of (positive) drift parameters,  $C$  is a constant  $2 \times 2$  lower triangular Choleski square root matrix of the covariance matrix  $V$ , and  $Z(t)$  is a  $2 \times 1$  vector of independent standard normal variables. Cairns et al. (2006) show that this model provides a good fit to UK Government Actuary’s Department (GAD) data for English and Welsh males over 1961–2002. For each set of parameter values, we simulated 10 000 paths of  $A_1(t + 1)$  and  $A_2(t + 1)$ , and then used these in (A.1) to obtain 10 000 simulated paths of  $q(t, x)$  over a chosen horizon.

Now let  $S(t, x)$  be the survival rate at time  $t$  of a cohort aged  $x$  in year 0. For any given  $x$ ,  $S(0, x) = 1$  and  $S(t, x)$  should

<sup>10</sup> Of course, in interpreting the fan chart forecasts, we also need to be on our guard against possible biases in the model: (1) the longevity forecasts have a possible downward bias in so far as they do not take account of future improvements due to medical science (e.g., miracle cures of major illnesses) that we cannot predict; (2) the forecasts have a possible upward bias in that they ignore important factors such as the impact of obesity that threaten to increase future mortality but have not yet fed through into the mortality data on which our model is calibrated. Readers who have strong views on these issues might wish to take them into account in interpreting the fan charts.

<sup>11</sup> More details of the algorithms used to generate Table 2 are provided in Dowd et al. (2007).

<sup>12</sup> Book of Genesis 5:27: ‘And all the days of Methuselah were nine hundred and sixty and nine years: and he died.’

Table 2  
Projected future annuity prices

Age	Current fair annuity price	Horizon (years)			
		$T = 5$	$T = 10$	$T = 15$	$T = 20$
(a) Results assuming all interest rates = 4% both now and at future time $T$					
		% Change in expected future annuity price at time $T$ relative to current annuity price			
55	17.2	3.8	5.7	7.4	8.9
65	12.9	5.6	8.4	11.0	13.3
75	8.3	7.5	11.2	14.7	18.0
		Central 90% prediction interval for % rate of change in projected future annuity price relative to current annuity price			
55		[1.12, 6.3]	[2.0, 8.8]	[2.8, 11.1]	[3.5, 13.3]
65		[5.6, 9.9]	[1.6, 14.0]	[2.3, 17.8]	[3.3, 21.4]
75		[-1.5, 15.3]	[-1.2, 21.2]	[-1.0, 26.9]	[-0.2, 32.7]
(b) Results assuming all current interest rates = 4% and all interest rates at time $T = 3%$					
		% Change in expected future annuity price at time $T$ relative to current annuity price			
55	17.2	18.3	20.6	22.8	24.8
65	12.9	16.5	19.9	23.0	25.9
75	8.3	15.2	19.4	23.3	27.1
		Central 90% prediction interval for % rate of change in projected future annuity price relative to current annuity price			
55		[14.8, 21.5]	[15.7, 24.9]	[16.7, 27.9]	[17.6, 30.8]
65		[10.6, 21.8]	[11.6, 26.9]	[12.4, 31.4]	[13.6, 35.9]
75		[5.0, 24.0]	[5.3, 30.8]	[5.4, 37.3]	[6.2, 43.9]

As per the Notes in Table 1 pertaining to the parameter uncertain model. All annuity prices are calculated as the net present value of the expected survivor payments predicated on a 10% loading factor.

diminish as  $t$  gets bigger and eventually go to 0 as  $t$  gets very large. Given any path of  $q(t, x)$  as obtained above, we then obtain a corresponding path of  $S(t, x)$  from the relationship between mortality and survival rates, i.e., from

$$S(t + 1, x) = (1 - q(t, x))S(t, x). \tag{A.3}$$

For each given  $t$  and  $x$ , the quantiles of  $S(t, x)$  were obtained from the relevant order statistics of our ‘sample’ of  $S(t, x)$  values, and these quantiles give us the bounds of the fan chart intervals.

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