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Paul Sweeting

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The Pensions Institute
Cass Business School
City University
106 Bunhill Row London
EC1Y 8TZ
UNITED KINGDOM

http://www.pensions-institute.org/

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Dr Paul Sweeting PhD FIA CFA

Pensions Institute, Cass Business School, City University, London

paul.sweeting.1@city.ac.uk; mail@paulsweeting.co.uk

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Abstract

This paper builds on the two-factor model developed by Cairns *et al* (2006) for projecting future mortality. It is shown that these two factors do not follow a random walk, as proposed by Cairns *et al*, but should instead be modeled as a random fluctuation around a trend, the trend changing periodically. Projecting mortality rates in this way suggests much greater uncertainty over future mortality improvements.

Keywords

stochastic mortality; mortality projections; mortality trends; Lee-Carter;

1. Introduction

In recent years, there have been a number of models developed to try to ascertain future changes in mortality rates. The most well-known of these is the model developed by Lee and Carter (1992), but this has seen many developments, such as the more robust treatment of errors by Brouhns *et al* (2002) and the addition of cohort effects by Renshaw and Haberman (2003, 2006). There are also continuous time variants such as those developed by Milevsky and Promislow (2001), Dahl (2004), Dahl and Møller (2005), Miltersen and Persson (2005), Biffis (2005) and Schrager (2006). The version I consider, however, is a discrete time model developed by Cairns *et al* (2006), to which I refer as the CBD model. Whilst Cairns *et al* (2007) do develop more complex models, the CBD model's simplicity is attractive and better allows the demonstration of the approach in this paper.

The CBD model is an innovative two-factor model. It assumes that each of the two parameters follows a random walk with drift, the rate of drift being constant and changes in the parameters being correlated. This approach is well-suited for the pricing of mortality-related derivatives with a term of a few years; however, when considering the very long-term, it becomes clear that the patterns for these two factors do not necessarily resemble a random walk; for most periods each of the factors can be modeled as a random fluctuation around a trend, the trend changing periodically.

The CBD model describes the logit of the initial mortality rate with a slope term and an intercept term, allowing for the number of deaths to follow a poisson distribution. Future stochastic simulations are then obtained by projecting these two terms as following correlated random walks. In other words:

(1) $$\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k1_t + (x \times k2_t) + \varepsilon_{x,t}$$

where $q_{x,t}$ is the initial mortality rate for a life aged x at time t ; $k1_t$ is the intercept term, kappa 1, at time t ; $k2_t$ is the slope term, kappa 2, at time t ; and $\varepsilon_{x,t}$ is an error term. The CBD model is calibrated using data over the period 1961 to 2002. However, by extending the time period over which the CBD model is fitted to cover the period from 1841 to 2005, clear patterns can be seen in kappa 1 and kappa 2, as shown in Figure 1. The data used for these calculations is obtained from the Human Mortality Database (2008). Figure 1 suggests that both kappa1 and kappa2 exhibit variation around a trend rather than following random walks. They also suggest that these trends change suddenly and definitely, that changes in the trend for kappa 1 and kappa2 tend to happen at the same time, and that there is strong negative correlation between the direction of these trend changes.

[FIGURE 1 ABOUT HERE]

The organization of this paper is, then, as follows. First, I consider ways in which change-points in the trend can be determined. I then fit trends to different sections of the data. Within each of these sections, I then consider whether the observations are trend- or difference-stationary. Next, I propose an approach for projection mortality rates based on this structure, and finally I show the results of some of these projections.

2. Determining the Change-Points in the Trend

Looking at Figure 1, it is clear that there are breaks in the trend for both kappa 1 and kappa 2, despite the higher volatility at the start of the period under investigation. The most likely explanation for this difference in volatility is that the lower life expectancies early in the period meant that the number of lives at higher ages was smaller, leading to greater volatility in observed mortality rates and thus in the parameterisation of the model. Despite the clear existence of breaks, it is not necessarily clear exactly where these breaks occur, and whether any changes are significant enough to be considered changes in trend.

One way of investigating possible change-points is to fit lines to sections of kappa 1 and kappa 2 and to calculate the Durbin-Watson (DW) statistic. If the line fitted covers data within a single trend, then the DW statistic should show no evidence of significant serial correlation. However, if a line covers, say two periods where the rate of change is lower in the second period than the first, then the first and last sections of the data would lie below the line whilst the middle section would lie above it. This would lead to the DW statistic showing significant positive serial correlation. This approach is not exact, but gives some clues as to where breaks might occur.

The exact approach used is to fit a line for each year y where y =1841 to 2003 (so using at least three years of data to calculate the DW statistic) and, within each year for each period p where p =3 to 2005-y . This approach can point to broad areas where changes in trend might occur; however, when fitting a series of lines to this data, some trial and error is required to get a good fit.

3. Fitting a Model

The broad approach used to fit lines to kappa 1 and kappa 2 is a weighted least squares approach. This is necessary because of the heteroskedasticity present in both kappa 1 and kappa2 – the variances decrease substantially over time. The weights used are the reciprocals of the variances for the seven years centered on the observation in question. The weights for the first three observations are set equal to the fourth observation; the weights for the last three observations are set equal to the fourth from last observation.

I first consider kappa 1. For each variable, break points in the trend are identified. Let the final year of a trend (and the first year of the next) for kappa 1 be identified as $b_{k1,n(k1)}$ where $n(k1) = 1 \ldots N(k1) - 1$. This means that the number of break points is $N(k1) - 1$ and the number of lines to be fitted is $N(k1)$. Each line is expressed as a constant, $\alpha_{k1,n(k1)}$ plus a slope, $\beta_{k1,n(k1)}$, the latter being multiplied by the year. This means that the estimate of kappa 1, $\widehat{k1}$, is described as follows:

(2) $$\widehat{k1} = \begin{cases} \alpha_{k1,1} + \beta_{k1,1} y \text{ if } y \leq b_{k1,1} \\ \alpha_{k1,2} + \beta_{k1,2} y \text{ if } b_{k1,1} < y \leq b_{k1,2} \\ \ldots \\ \alpha_{k1,N(k1)} + \beta_{k1,N(k1)} y \text{ if } y > b_{k1,N(k1)-1} \end{cases}$$

Similarly, the estimate of kappa 2, $\widehat{k2}$, is described as follows:

(3) $$\widehat{k2} = \begin{cases} \alpha_{k2,1} + \beta_{k2,1} y \text{ if } y \leq b_{k2,1} \\ \alpha_{k2,2} + \beta_{k2,2} y \text{ if } b_{k2,1} < y \leq b_{k2,2} \\ \ldots \\ \alpha_{k2,N(k2)} + \beta_{k2,N(k2)} y \text{ if } y > b_{k2,N(k2)-1} \end{cases}$$

It can also be said that

(4) kappa 1 $= \widehat{k1} + \varepsilon_{k1}$, and

(5) kappa 2 $= \widehat{k2} + \varepsilon_{k2}$, and

where ε_{k1} and ε_{k2} are the residuals. Initially, $N(k1)$ is set equal to $N(k2)$. Furthermore, all $b_{k1,n(k1)}$ are set equal to $b_{k2,n(k2)}$. This second point seems sensible, since a visual inspection of kappa 1 and kappa 2 suggests that changes in trends occur at about the same time in one as in the other, although there are perhaps some instances when a significant change occurs in only one series. This is investigated below.

When fitting the lines, all potential changes in trend are considered initially. The lines are then fitted by minimizing the sum of squared errors subject to the restriction that:

(6) $$\alpha_{k1,n(k1)} + \beta_{k1,n(k1)} b_{k1,n(k1)} = \alpha_{k1,n(k1)+1} + \beta_{k1,n(k1)+1} b_{k1,n(k1)}$$

and:

(7) $$\alpha_{k2,n(k2)} + \beta_{k2,n(k2)} b_{k2,n(k2)} = \alpha_{k2,n(k2)+1} + \beta_{k2,n(k2)+1} b_{k2,n(k2)}$$

for $n(k1) = 1 \ldots N(k1)$ and $n(k2) = 1 \ldots N(k2)$. The first test I carry out is a Chow test. For each consecutive pair of trends, I also fit a single line covering both trends and calculate the test statistic as:

(8) $$CT = \frac{(SSR_T - (SSR_1 + SSR_2))/v}{(SSR_1 + SSR_2)/(N_1 + N_2 - 2v)} \sim F^{v}_{N_1+N_2-2v}$$

where SSR_1, SSR_2 and SSR_T are the sums of squared residuals from the first section of data, the second section of data and the combined dataset respectively. The number of variables, given by v is two, and the numbers of observations in the first and second groups of data are given by N_1 and N_2.

The restrictions in (6) and (7) effectively mean that the slope parameters are optimized subject to restrictions on the level parameters. I therefore look at the difference between successive slope parameters to identify where any change in slope is not statistically significant, calculating a t-statistic using the joint standard error calculated assuming unequal sample sizes and unequal variances in each sample.

I also consider the DW statistic for the various sections. If separate trends are combined, then this may be highlighted by significant positive serial correlation, suggesting that a change in trend has been missed.

The results are given in Table 1 for kappa 1 and Table 2 for Kappa 2. The first columns give the first year of the new trend and the last year of the previous one. The second and third columns give the intercept and slope parameters respectively, with standard errors for each variable in parentheses below.

The fourth column in each table gives the DW statistics for each trend period. The DW statistic is centered on 2. When considering positive serial correlation, a statistic with a value less than dL gives evidence of significant positive serial correlation, one with a value greater than dL gives no evidence of significant positive serial correlation, and one with a value between dL and dU suggests only possible positive serial correlation. This is designed to show whether combining lines has led to a single fit covering more than one trend (as would be seen with significant positive serial correlation).

The fifth column gives the results of Dickey-Fuller (DF) tests for each section of the data. Under the null hypothesis of a DF test, the time series under investigation follows a random walk with drift – in other words, it has a unit root. The alternative hypothesis is that the time series consists of random deviations from a trend.

The sixth column gives the difference in the slope parameter from period to period, together with the standard error of the difference in parentheses below. The asterisks denote whether the difference between the trends is statistically significant.
Finally, the seventh column in each table gives the Chow test results.

[TABLE 1 ABOUT HERE]

[TABLE 2 ABOUT HERE]

Looking first at kappa 1, a number of break points appear to be less than convincing, suggesting the need for further investigation; however, the only break point not strongly identified by the Chow tests in kappa 2 is the first.

Trying various combinations of the trends and examining the various statistics suggests that the breakpoints identified in Tables 3 and 4, below, better describe the data.

[TABLE 3 ABOUT HERE]

[TABLE 4 ABOUT HERE]

Again, looking first at kappa 1, the Chow test statistic strongly suggests breaks at all the points remaining. However, the change-in-trend test also suggests changes in all but one of these instances, and further combining the trends in the one instance where no change is suggested results in significant positive serial correlation. In fact, one of the combinations carried out has also resulted in a single case of positive serial correlation; however, looking at the results in Figure 2 suggest that this is more likely to be a result of random clustering than two separate trends having been combined.

[FIGURE 2 ABOUT HERE]

The single change in kappa 1 is less controversial – the Chow test and the change in trend analysis both suggest that no further changes are needed, and the DW statistics do not suggest that any trends have been combined incorrectly.

One reported statistic has not been discussed in detail: the DF statistic. For kappa 1, the DF test suggests that the data follows random deviations around a trend in six out of eight periods; for kappa 2, the figure is seven out of nine periods. In most case, the level of significance is 1%.

4. A Projection Approach

Having fitted a model to the data, I next look at some of the parameters with a view to using this data to project mortality forward.

The trend in kappa 1 changes 7 times out of a possible 164 times, a probability of 0.036585; the trend in kappa 2 changes 8 times in the same period, a probability of 0.054878. However, kappa 1 and kappa 2 change together only six of these times. These changes can therefore be modeled by simulating a uniform random variable, $0 \le \Delta_k < 1$, such that:

- if $0 \le \Delta_k <$ 0.006098 then kappa 1 only changes;
- if $0.006098 \le \Delta_k <$ 0.042683 then kappa 1 and kappa 2 change;
- if $0.042683 \le \Delta_k <$ 0.054878 then only kappa 2 changes; and
- if $\Delta_k \ge$ 0.054878 then neither kappa 1 nor kappa 2 change.

The next stage is to look at the variation in kappa 1 and kappa 2 when they change. One approach would be to consider the root mean square (RMS) of deviations and use these to define the volatility. This would mean essentially assuming that the expectation was for the current trends in kappa 1 and kappa 2 to continue indefinitely, with each being as likely to accelerate as to decelerate. The alternative is to use the standard deviation. This would give a lower level of volatility, but would require an assumption that changes in kappa 1 and kappa 2 would be expected to accelerate or decelerate (depending on whether the average was positive or negative). This latter view seems unrealistic, as it suggests that both kappa 1 and kappa 2 would eventually tend towards positive or negative infinity. I therefore use the RMS approach. I also calculate a measure of correlation between kappa 1 and kappa 2 when both change using a similar approach, calculating a measure of covariance based on deviations from zero rather than from the mean, then dividing the result by the RMS of kappa 1 and kappa 2. The correlation calculated by this method is -0.301142, compared with the “true” correlation of 0.380513. Given that the two series appear to move in opposite directions, the negative correlation seems more plausible.

When both kappa 1 and kappa 2 change, the RMS volatility of the former is 0.008406 (compared with a standard deviation of 0.005597) and of the latter is 0.000405 (compared with a standard deviation of 0.000352). However, the RMS volatility for kappa 1 using all changes is 0.015998 (compared with a standard deviation of 0.013719), and for kappa 2 is 0.000332 (compared with a standard deviation of 0.000287). An F-test suggests that the volatility differs for kappa 1, but only at the 10% level; there is no significant difference for kappa 2. However, the small number of observations means that it is difficult to draw any strong conclusions. In the analysis below, I use the same volatility when only one variable changes or one both change.

Finally, given that the data is to be modeled assuming random volatility around a trend, the nature of this volatility needs to be investigated. Analysis of the standard deviation of the errors shows that the volatility decreases successively in each period, but there is a particularly large and sustained fall for both kappa 1 and kappa 2 in 1973. I therefore assume that the volatility around the trend is given by the volatility calculated using data since this date, 0.012098 for kappa1 and 0.000627 for kappa 2.

5. Projection Results

Using the method and data above, I carry out 1,000 simulations of kappa 1 and kappa 2. I then use these to calculate the period life expectancy of a 60-year old male for the fifty year period from 2006 to 2056. I show the results in Figure 3, below. Displayed are the median and various percentile limits, together with three sample paths.

[FIGURE 3 ABOUT HERE]

This shows that the range of results in early years is relatively narrow, but uncertainty does grow rapidly. To illustrate, the difference between the 5th and the 95th percentiles for period life expectancy in 2056 is 18.7 years. Dowd *et al* (2008) perform similar calculations to arrive at cohort (rather than period) life expectancies for 65 year old males for the same projection period (2006 to 2056), using a version of the CBD model that allows for cohort effects. If parameter uncertainty is ignored, the 90% confidence interval in 2056 spans 3.6 years; even if parameter uncertainty is allowed for, the range rises to only 7.6 years. In other words, this trend-change model suggests more than twice as much uncertainty as the cohort-adjusted CBD model with parameter uncertainty over a fifty-year time horizon, and the shape of the funnel suggests that the difference in uncertainty continues to increase. This is because the effect of a single change in the trend increases over time. Furthermore, an offsetting change in the trend at some point in the future would not bring the parameter back to its original value, meaning that the overall level of uncertainty grows rapidly when looking at long time horizons. This is not the case for a random walk, where the trend is unaffected by random variation.

6. Conclusion

If a two parameter model of the type described by Cairns *et al* (2006) is used to model mortality, then the parameters follow clear trends that change periodically. The changes in the parameters frequently occur at the same time and are negatively correlated. Within each trend, the parameters do seem to be random fluctuations around a trend rather than random walks. Modeling mortality this way into the future suggests a much greater degree of uncertainty than some other models.

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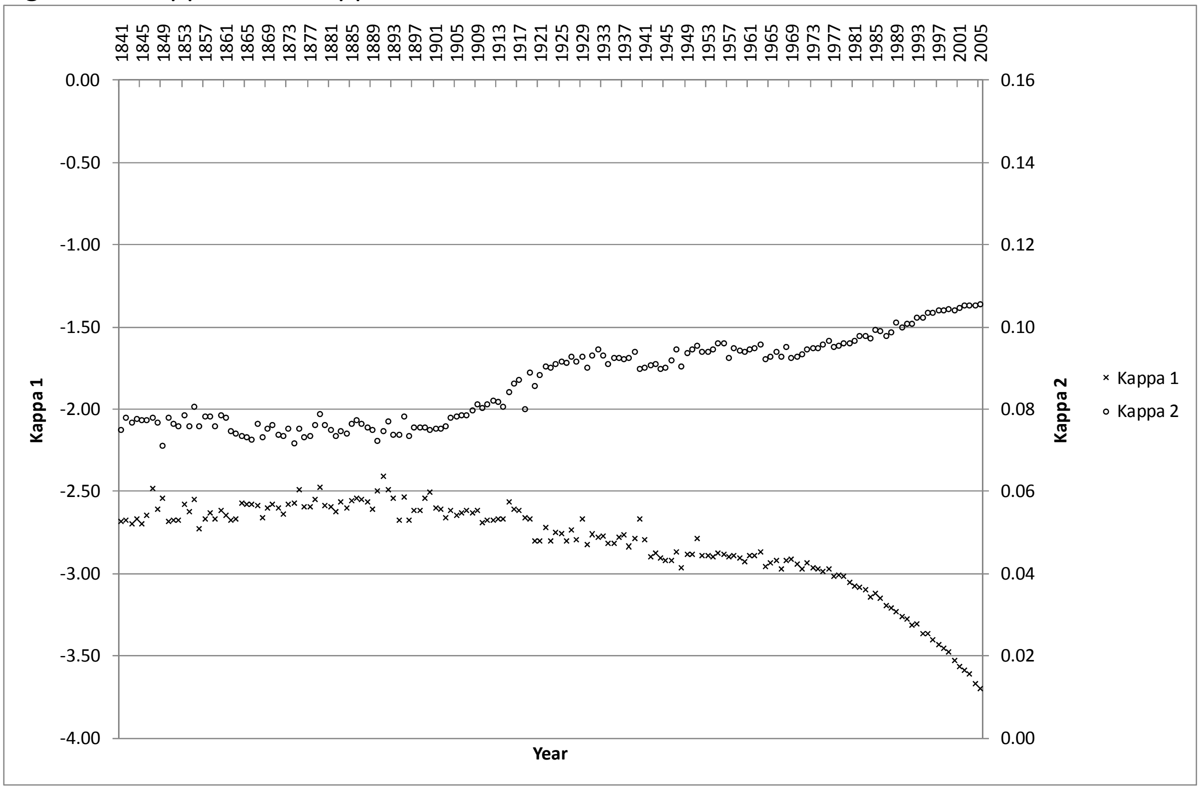
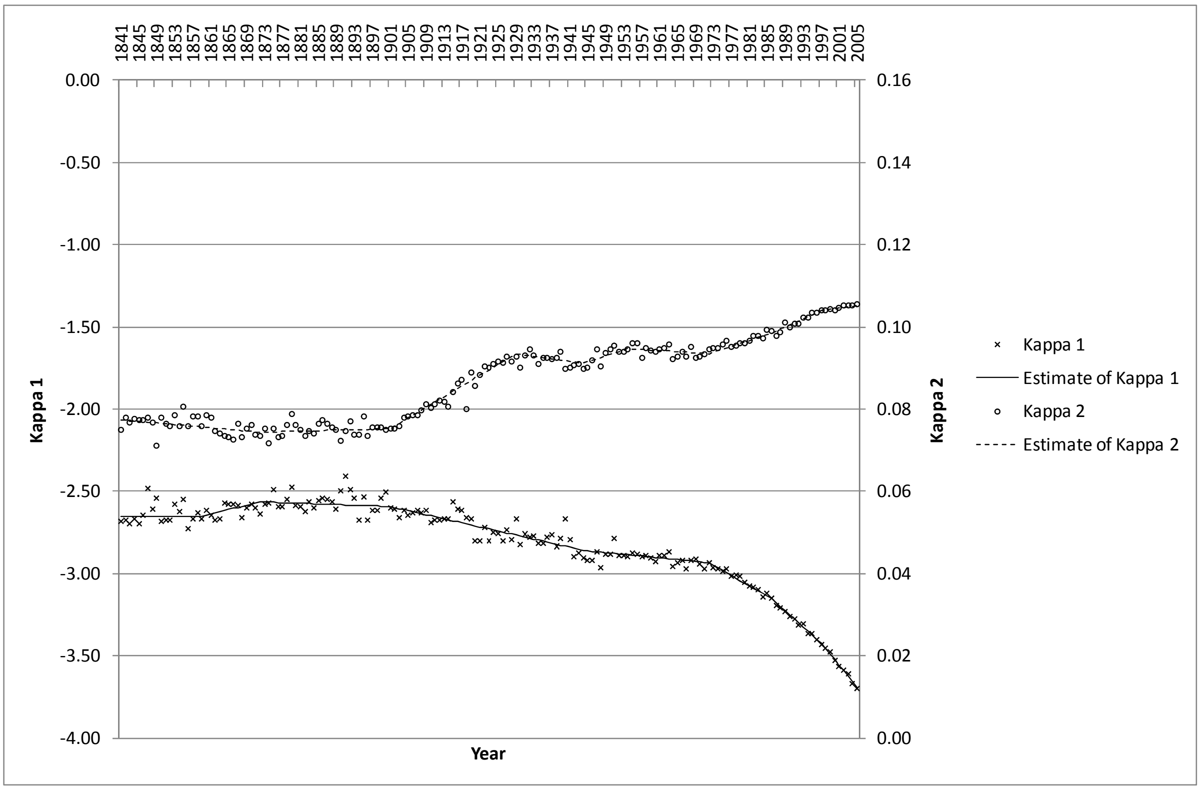
Tables and Figures

Figure 1 – Kappa 1 and Kappa 2 for the CDB Model, 1841-2005



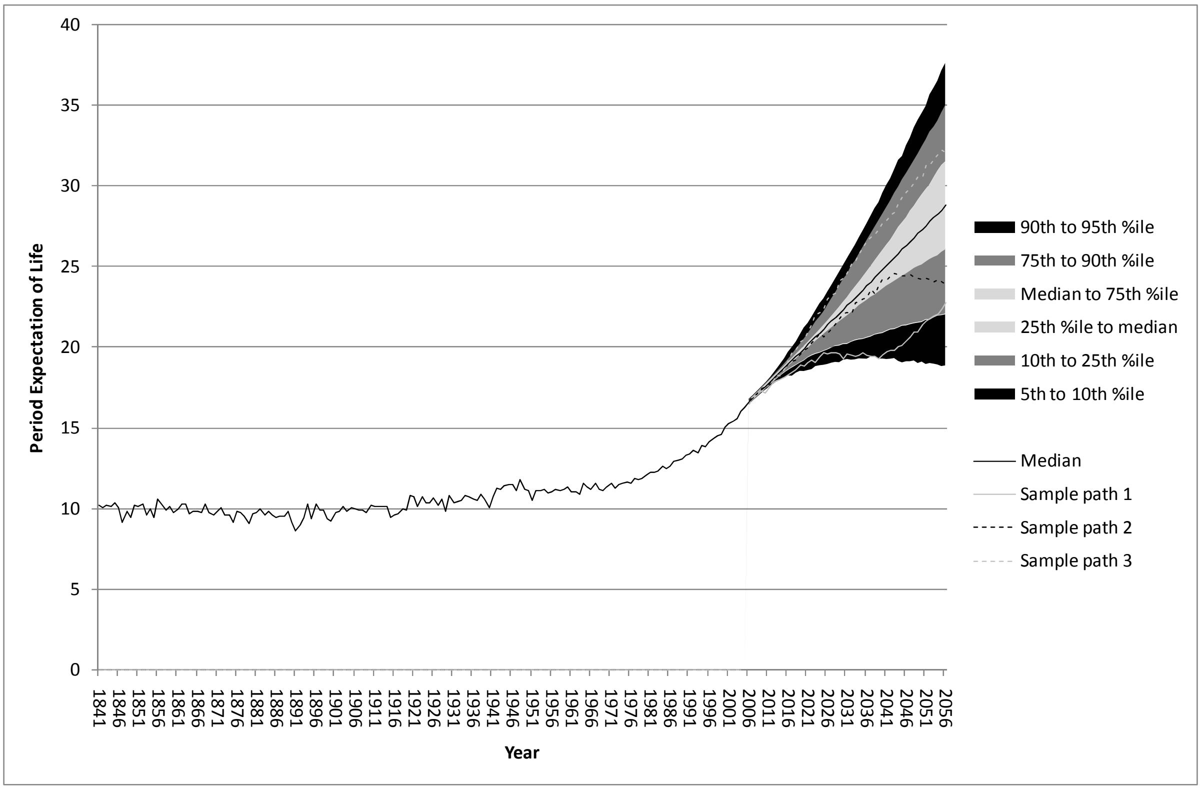
Source: Human Mortality Database (2008), Author's Calculations

Figure 2 – Kappa 1, Kappa 2 and Fitted Lines, 1841-2005



Source: Human Mortality Database (2008), Author's Calculations

Figure 3 – Historical and Projected Period Life Expectancy for 65 Year Old Males, England and Wales, 1841-2056



Source: Human Mortality Database (2008), Author's Calculations

Table 1 – Parameters for Fitted Values of Kappa 1

First Year of Trend	Kappa 1 Intercept (SE)	Kappa 1 Slope (SE)	DW Statistic		DF Statistic		Δ Kappa 1 Slope (SE)		Chow Test Statistic	
1841	-2.96683	0.000168	1.701532		-3.518565	*				
	(4.994174)	(0.002700)					0.006502	**	3.694137	**
1860	-15.0547	0.006671	1.219745	+	-2.279956		(0.002856)			
	(5.491361)	(0.002943)					-0.007657	***	2.668865	*
1873	-0.72055	-0.000986	1.496792		-4.129292	**	(0.002069)			
	(2.678539)	(0.001419)					-0.005125	***	13.189975	***
1902	9.021928	-0.006111	1.256072	++	-3.055781		(0.001407)			
	(2.679251)	(0.001399)					-0.001805		0.355020	
1930	12.50438	-0.007917	1.042455	++	-2.349303		(0.002875)			
	(7.156921)	(0.003695)					0.008425		5.760253	**
1945	-3.87419	0.000509	1.974323		-2.828847		(0.006203)			
	(15.239244)	(0.007817)					-0.003804		3.221651	*
1955	3.558806	-0.003295	1.950550		-4.133678	**	(0.005403)			
	(2.530577)	(0.001289)					-0.011955	***	26.251961	***
1973	27.13348	-0.015250	1.784664		-3.667513	**	(0.001123)			
	(1.875906)	(0.000948)					-0.009425	**	23.103952	***
1988	45.85149	-0.024675	2.584364		-5.415041	***	(0.003682)			
	(10.024917)	(0.005033)					-0.008391		48.665600	***
1998	62.6076	-0.033066	1.951402		-2.331239		(0.005222)			
	(10.800210)	(0.005396)								

Significance level: *** 1%; ** 5%; * 10%.
Serial correlation: ++ evidence of significant positive serial correlation (statistic <dL); + evidence of possible positive serial correlation (dL<statistic<dU).

Table 2 – Parameters for Fitted Values of Kappa 2

First Year of Trend	Kappa 2 Intercept (SE)	Kappa 2 Slope (SE)	DW Statistic		DF Statistic		Δ Kappa 2 Slope (SE)		Chow Test Statistic	
1841	0.019917	0.000031	2.503566		-5.360380	***				
	(0.150937)	(0.000082)					-0.000344	**	1.727529	
1860	0.658869	-0.000313	1.148115	+	-2.600994		(0.000126)			
	(0.272553)	(0.000146)					0.000385	***	12.562046	***
1873	-0.06094	0.000072	1.535067		-4.872952	***	(0.000092)			
	(0.080100)	(0.000042)					0.000574	***	94.695468	***
1902	-1.15299	0.000646	2.402800		-6.538536	***	(0.000041)			
	(0.078094)	(0.000041)					-0.000800	***	76.076018	***
1930	0.389713	-0.000154	1.348259	+	-4.161198	**	(0.000073)			
	(0.178596)	(0.000092)					0.000512	***	40.616383	***
1945	-0.60562	0.000358	2.369726		-3.530067	*	(0.000141)			
	(0.339579)	(0.000174)					-0.000430	***	85.953783	***
1955	0.234761	-0.000072	2.138283		-4.409199	***	(0.000125)			
	(0.108824)	(0.000055)					0.000430	***	29.847510	***
1973	-0.61336	0.000358	1.178452	+	-2.773581		(0.000056)			
	(0.110733)	(0.000056)					0.000141	**	11.426100	***
1988	-0.89299	0.000499	2.166787		-4.649186	***	(0.000064)			
	(0.142363)	(0.000071)					-0.000346	***	69.007492	***
1998	-0.20282	0.000154	1.330504	+	-2.244186		(0.000063)			
	(0.107361)	(0.000054)								

Significance level: *** 1%; ** 5%; * 10%.
Serial correlation: ++ evidence of significant positive serial correlation (statistic <dL); + evidence of possible positive serial correlation (dL<statistic<dU).

Table 3 – Revised Parameters for Fitted Values of Kappa 1

First Year of Trend	Kappa 1 Intercept (SE)	Kappa 1 Slope (SE)	DW Statistic		DF Statistic		Δ Kappa 1 Slope (SE)		Chow Test Statistic	
1841	-2.96683	0.000168	1.696690		-3.518565	*				
	(4.994174)	(0.002700)					0.006537	**	5.133120	***
1860	-15.12	0.006706	1.220154	+	-2.279956		(0.002856)			
	(5.491361)	(0.002943)					-0.007700	***	8.041144	***
1873	-0.70505	-0.000995	1.495064		-4.129292	**	(0.002069)			
	(2.678539)	(0.001419)					-0.005182	***	16.802856	***
1902	9.145043	-0.006176	1.447660	++	-4.727278	***	(0.001046)			
	(1.434794)	(0.000746)					0.003530	***	6.216409	***
1945	2.28253	-0.002646	1.673548		-4.705058	***	(0.000809)			
	(1.673403)	(0.000854)					-0.012966	***	33.455055	***
1973	27.8521	-0.015612	1.885997		-3.667513	**	(0.000992)			
	(2.180101)	(0.001101)					-0.008897	**	100.169916	***
1988	45.52208	-0.024510	2.629184		-5.415041	***	(0.003612)			
	(9.768575)	(0.004904)					-0.008834		32.807038	***
1998	63.16286	-0.033343	2.030812		-2.331239		(0.005106)			
	(10.588903)	(0.005290)								

Significance level: *** 1%; ** 5%; * 10%.

Serial correlation: ++ evidence of significant positive serial correlation (statistic <dL); + evidence of possible positive serial correlation (dL<statistic<dU).

Table 4 – Revised Parameters for Fitted Values of Kappa 2

First Year of Trend	Kappa 2 Intercept (SE)	Kappa 2 Slope (SE)	DW Statistic		DF Statistic		Δ Kappa 2 Slope (SE)		Chow Test Statistic	
1841	0.255659	-0.000097	1.863737		-5.240416	***				
	(0.069743)	(0.000038)					0.000127	***	3.351952	**
1873	0.017483	0.000030	1.903972		-6.725793	***	(0.000038)			
	(0.071755)	(0.000038)					0.000619	***	109.848615	***
1902	-1.159045	0.000649	2.419991		-2.656571	***	(0.000040)			
	(0.078098)	(0.000041)					-0.000803	***	71.745047	***
1930	0.389737	-0.000154	1.331124	+	-3.071266	**	(0.000074)			
	(0.181001)	(0.000093)					0.000512	***	40.299153	***
1945	-0.605591	0.000358	2.377781		-3.264308	*	(0.000141)			
	(0.338518)	(0.000174)					-0.000430	***	79.795420	***
1955	0.234785	-0.000072	2.133573		-3.613035	***	(0.000125)			
	(0.109126)	(0.000056)					0.000430	***	30.105090	***
1973	-0.613332	0.000358	1.215524	+	-3.819187		(0.000055)			
	(0.107336)	(0.000054)					0.000141	**	7.237369	***
1988	-0.892961	0.000499	2.105288		-3.912770	***	(0.000065)			
	(0.146787)	(0.000074)					-0.000346	***	58.677171	***
1998	-0.202798	0.000154	1.414360		-3.978984		(0.000063)			
	(0.101564)	(0.000051)								

Significance level: *** 1%; ** 5%; * 10%.

Serial correlation: ++ evidence of significant positive serial correlation (statistic <dL); + evidence of possible positive serial correlation (dL<statistic<dU).