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# Portfolio Choice Models of Pension Funds and Life Assurance Companies: Similarities and Differences

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# Portfolio Choice Models of Pension Funds and Life Assurance Companies: Similarities and Differences*

by David Blake**

In this paper, we examine pension schemes and life policies in terms of the option features either implicitly or explicitly contained in them. We argue that this greatly simplifies the process of managing the asset side of pension fund and life company balance sheets. We show that these options need to be either replicated or hedged with an appropriately determined portfolio of cash market and derivative securities. The natural portfolio choice framework for achieving this is asset-liability or surplus risk management. We use this framework to derive a dynamic asset allocation strategy over the lifetime of a pension scheme which involves switching, according to pre-set criteria, between equities, index bonds and conventional bonds. We also use this framework to derive a procedure for hedging the risk faced by life companies from the exercise of the options embodied in their products. We end the paper with an example that applies our dynamic asset allocation strategy to the case of a defined benefit pension scheme.

Pension funds and life assurance companies are the principal long-term investing institutions and as such they will have liabilities of the longest duration. These liabilities will also be similar in nature, although there will be qualitative differences, e.g., life policies provide for such features as policy loan and early surrender options in a way that pension funds do not, and defined benefit pension schemes have options on the invested assets in a way that life policies do not. The greatest systematic risk faced by both sets of institutions arises from mismatches of various kinds between assets and liabilities. For example, to minimize the risks associated with maturity mismatching, the two sets of institutions will tend to hold a substantial proportion of long-term assets, such as equities, property and long-term bonds, in their portfolios; although, given the specific nature of the options attached to life policies, life companies will hold a relatively larger proportion of more capital-certain assets, such as bonds, in their portfolios than pension funds.

## 1. Introduction

Pension funds and life assurance companies comprise the principal long-term investing institutions in any economy with an advanced financial system. Occupational pension schemes have been around since at least the 14th century, but life assurance companies are older than pension funds. The world's oldest life company is the Equitable Life Assurance Society which began selling life assurance and annuities in 1762, following the development of premium tables based on actuarial calculations. Pension funds, by comparison, are essentially a 20th-century phenomenon: they began to grow slowly after World War I,

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beginning in countries such as the U.K., the U.S. and Holland. While there are some large unfunded occupational pension schemes in the U.K. public sector, most U.K. private occupational schemes are funded. However, the vast majority of the 190,000 private occupational pension schemes are very small and these small schemes are managed and assured with life companies on a group-life basis. The first British insurance company to offer pension fund assurance was Legal and General in the 1930s. Larger companies administer their own pension schemes and tend to appoint their own fund managers or to have their scheme assets managed by externally-appointed fund managers such as Mercury Asset Management or Gartmore Fund Managers. Insurance companies are the largest providers of personal pension schemes (i.e., individual defined contribution schemes) which first became available in the U.K. in 1988. While other financial institutions, such as banks, building societies, unit and investment trusts, and open-ended investment companies (OEICs) can also sell personal pensions, the resulting pension annuity must be purchased from a life company.

Although pension funds and life companies are separate legal entities, there is clearly a substantial overlap between their activities. The purpose of this paper is to examine the differences, and, more importantly, the similarities, between pension funds and life companies and the consequences these have for the portfolio choice models adopted by these institutions.

## 2. The similarities and differences between pension funds and life assurance companies

Pension funds and life companies have very similar objectives. The objective of a life company has been stated as being “to maximize return, subject to risk, within an asset structure that takes the term structure of its liabilities into account” (evidence presented in 1977 to the Wilson Committee, 1980). The objective of a pension fund has been stated as being “to invest the scheme monies in such a way as to ensure that the scheme will always have resources available to meet its liabilities to pay benefits as and when they fall due at all times in the future; and, in so doing, to take account of the risk factors inherent in any investment situation” (Martin and Grundy, 1987, p. 3).

However, there are some important differences between life companies and pension funds. First, life companies are profit-making organizations owned by shareholders, whereas pension funds are non-profit-making organizations without shareholders.[1] Second, life companies are in the spread business. Life companies, like banks, earn a reward from the spread between the return on their assets and the cost of the funds they raise in the market; another objective is to maximize the net underwriting margin (defined as (Premium income – Policy expenses)/Total assets)). In contrast, pension funds are not in the spread business, since they do not directly raise funds in the market. Rather, they seek to meet their pension obligations at minimum cost to the scheme sponsor. Furthermore, whilst both are long term in nature, the liabilities of life companies and pension funds are nevertheless qualitatively different, although they still have much more in common than the liabilities of, for example, banks which operate at the short-term end of the financial system. These differences arise from the different range of products offered by the two sets of institutions.

[1] Some providers of life assurance are mutual societies, which are not incorporated and do not make profits as such. They are owned by their members rather than shareholders, but they have the same essential objectives as profit-making life companies, namely the maximization of a surplus.

Life companies provide the following range of products, and these products constitute the life companies' liabilities:

(1) *Term assurance* – in exchange for a fixed premium, provides a fixed cash payout if death occurs during a specified term (and nothing otherwise);
(2) *Endowment assurance* – provides a fixed cash payout if death occurs during a specified term, and, if the policyholder survives the term, a cash payout that is either fixed at the start of the policy (in the case of a *non-profit* endowment policy) or depends on the performance of the assets purchased with the premiums (in the case of a *with-profits* or a *unit-linked* endowment policy); if the policy is terminated before the end of the term, the policy has an early *surrender value* which is generally much lower than the *maturity value*;
(3) *Whole life assurance* – provides a cash payout on the death of the policy holder; the policy has an early *surrender value* and also a *policy loan value*, since the life company is prepared to provide a loan against the policy as surety at a rate of interest known as the *crediting rate*;
(4) *Personal pensions* – provides a pension annuity depending on the size of the fund generated from the investment of contributions into the scheme;
(5) *Annuities* – in exchange for a single lump-sum premium, provides regular periodic cash flows for a term that is either fixed or is contingent upon the longevity of one or more persons. The most common annuity is the single-life annuity which ceases on the death of the annuitant; one variation provides a minimum guarantee, whereby the payments continue for a minimum fixed period whether or not the annuitant survives this period, while another variation provides capital protection, whereby the balance of capital is paid to the annuitant's estate when he or she dies. In the case of married couples, it is possible to take out a joint-survivor annuity which continues until the second death. All these annuities come with the following variations: level annuity, escalating annuity, unit-linked, with-profits, or index-linked. Finally, annuities can be differentiated according to whether they come into effect as soon as the single premium is paid (immediate annuities) or at a later date (deferred annuities).

As mentioned in the introduction, most private sector pension schemes in the U.K. are funded. A funded pension scheme is composed of a pension fund *plus* a pension annuity. Contributions from scheme members and (possibly) the scheme sponsor are made into the fund. The pension fund manager invests these contributions in assets which generate income and capital gains. Over time, a fund of financial assets accumulates. When the member retires, he or she receives a pension annuity; in some cases, this is financed by drawing down the fund of accumulated assets, while in others, the fund manager transfers to a life company a sum of money which is used to purchase the annuity. Typically, part of the annuity can be commuted as a lump sum on the retirement date; and usually the annuity is indexed, at least partially, to inflation. The annuity will continue until the death of the member and usually also until the death of his or her spouse or other dependant.[2]

What essentially differentiates one pension scheme from another is the set of rules relating to the way in which benefits are calculated when the scheme member retires. There are three main types of scheme and the obligations accumulating under each of these schemes constitute the liabilities of the scheme's sponsor or manager:

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[2] Details of the different types of pension schemes operating in the U.K., the E.U. and the U.S. are contained, respectively, in Blake (1995), Harrison (1995), Bodie and Shoven (1983), Bodie, Shoven and Wise (1987, 1988), and McGill *et al.* (1996).

(1) The simplest scheme is the *defined contribution ("DC") scheme* (otherwise known as a *money purchase* or a *personal pension scheme*): this uses the full value of the fund's assets to determine the amount of pension. The DC pension depends only on the value of this fund which, depending on the success of the fund manager, might be high or low. As stated above, life companies are the largest providers of these schemes;
(2) The *defined benefit ("DB") scheme* (otherwise known as a *salary-related scheme*), in contrast, calculates the benefit in relation to factors such as the final salary, the length of pensionable service, and the age of the member, rather than to the value of the assets in the fund as such. For example, a typical U.K. scheme provides a pension equal to one-sixtieth (1.67 per cent) of final salary for each year of pensionable service up to a maximum of 40 years' service; thus the maximum pension is two-thirds of final salary. If a DB scheme is guaranteed by a life company, it is known as an *insured scheme*, or if it is operated directly by the sponsor, it is known as a *non-insured* or *self-insured scheme;*
(3) In the case of a *targeted money purchase ("TMP") scheme*, the aim is to use a defined contribution scheme to target a particular pension at retirement (which may be the same as that resulting from a final salary scheme), but which also benefits from any upside potential in the value of the fund assets above that required to deliver this target level. In other words, the TMP scheme aims to provide a minimum pension but not a maximum pension. In the U.S., such schemes are known as designer pension schemes or floor-offset schemes.

The different products offered by life companies and pension funds generate different types of risks and these risks will be hedged with different portfolios of assets. In the case of life companies, for example, one risk is that the guaranteed sum to be paid at the end of the contract exceeds the premiums collected over the life of the contract. The life company will determine the size of the guaranteed sum based on actuarial assumptions about mortality and forecasts about returns on premiums invested. If these assumptions and forecasts turn out to be incorrect, then the value of the assets may be insufficient to meet the liabilities. The insurance company can minimize the risk of this arising by matching the maturity of its assets and liabilities. This, in turn, will help to immunize the company's exposure to interest rate risk, the risk that changes in interest rates along the yield curve adversely affects the surplus, the difference between the values of the assets and liabilities; the technique of immunization is discussed in detail in section 5 below. In the case where the future sum assured is fixed in nominal terms, interest rate risk is minimized by investing in fixed-income securities (especially government bonds) with the same maturity as the sum assured. Table 1 shows that insurance companies in the U.K. hold around 26 per cent of their assets in fixed-income securities. However, most policies written by insurance companies are with-profit policies where the policyholder receives both annual and terminal bonuses based on the investment performance of the invested assets. In such cases, the premiums will be invested in higher yielding but riskier assets, such as equities and property: Table 1 shows that insurance companies hold 59 per cent of their assets in equities (including unit trusts) and 7 per cent in property. Insurance companies must also ensure that they keep sufficient liquid funds to pay the lump sums on early surrenders and to meet any demands for policy loans: they hold 5 per cent of their assets in short-term securities to meet these contingencies. The official statistics do not record any holdings of derivative securities, such as financial futures and options.

In contrast, the liabilities of pension funds are generally more uncertain than those of life companies. The vast majority of pension schemes offer final-salary pensions that are subsequently indexed to inflation (at least partially). The risks faced by pension funds in delivering such pensions are much greater than the risks faced by life companies in delivering

*Table 1:*
*Asset allocation of U.K. life assurance companies and pension funds, end 1996 (%)*

| | **Life assurance companies** | **Pension funds** |
|---|---|---|
| Short term (net) | 5.1 | 4.6 |
| U.K. government bonds | 16.6 | 10.6 |
| (of which index-linked) | (2.0) | (5.0) |
| Other fixed income[a] | 7.2 | 1.2 |
| U.K. company shares | 40.6 | 50.7 |
| Unit trusts | 7.9 | 4.0 |
| Overseas fixed income[b] | 1.9 | 3.1 |
| Overseas company shares | 10.8 | 15.5 |
| Property (incl. unit trusts) | 6.6 | 4.5 |
| Loans and mortgages | 1.2 | 0.0 |
| Other (net) | 2.1 | 5.8 |
| Total assets (£bn) | 549.7 | 543.9 |

*Notes*:
[a]U.K. local authority and corporate bonds.
[b]Overseas government and corporate bonds.
*Source*: *Financial Statistics*, Office for National Statistics, September 1998, Tables 5.1A and 5.1B.

the guaranteed sum assured under life policies. Pension fund actuaries have to make forecasts of wage inflation up to retirement and price inflation after retirement. Not only will pension funds seek to match the maturities of assets and liabilities, they will need to do so with assets that generate positive real returns in the long term. Table 1 shows that pension funds hold 70 per cent of their assets in equities (including unit trusts) and 5 per cent in property, the principal "real" assets. In order to meet their near-maturing pension obligations which can be determined with greater certainty, pension funds also hold bonds. But the weighting of bonds in their portfolios is much lower than is the case with life companies, 15 per cent as against 26 per cent. Furthermore, a much higher proportion of the bonds are held in index-linked gilts: 33 per cent as against 8 per cent with life companies. Again holdings of derivative securities are not reported.

So recognizing the qualitatively different nature of the liabilities of life companies and pension funds, we can see how this translates into different portfolio asset allocations: because the liabilities of pension funds are more uncertain in both amount and timing and also more indexed to inflation than those of insurance companies, pension funds will hold a higher proportion of equities and indexed bonds in their portfolios; because life companies have to prepare for the possibility of early surrenders and policy loans, they will hold a higher proportion of bonds in their portfolios. But apart from this, the investment objectives of the two sets of institutions are remarkably similar and the investment management techniques pursued by the two sets of institutions will also be remarkably similar.

In section 3, we examine in more detail the option compositions of pension schemes and life policies. In section 4, we discuss how these options are valued. In section 5, we show how these option compositions can be utilized to develop strategies both for managing the assets of pension funds and insurance companies over time and for hedging the risks from early exercise of the options. In section 6, we illustrate the optimal dynamic asset allocation strategy using hypothetical data for a defined benefit pension scheme, and we draw conclusions in section 7.

## 3. The option composition of pension schemes and life assurance policies

Blake (1998) has examined the option composition of the three types of pension scheme considered in the last section. Figure 1 shows that the present value of the DC pension on the retirement date depends entirely on the value of the fund assets on that date. Figure 2 shows that the present value of the DB pension ($L$) is independent of the value of the fund assets, while Figure 3 shows that the TMP pension has a minimum present value of $L$, but is higher if asset values exceed $L$.

Figure 4 shows that the DB pension can be replicated using a long put option ($P$) and a short call option ($-C$) on the underlying assets of the fund ($A$), both with the same exercise price ($L$). The put option is held by the scheme member and written by the scheme sponsor, while the call option is written by the member and held by the sponsor. On the retirement date of the member, which coincides with the expiry date of the options, one of the options is

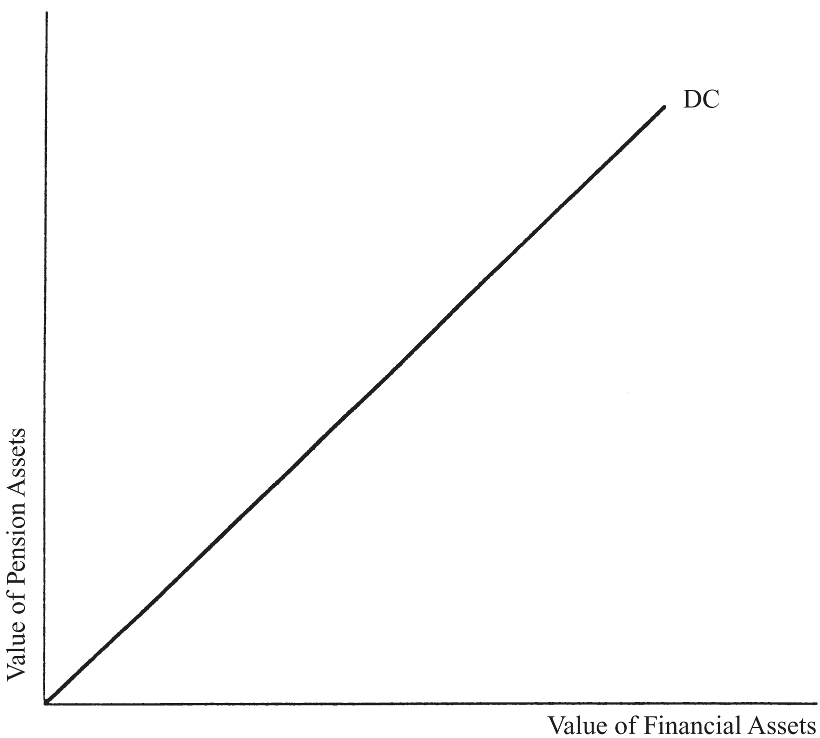


*Figure 1: A Defined Contribution Pension Scheme*

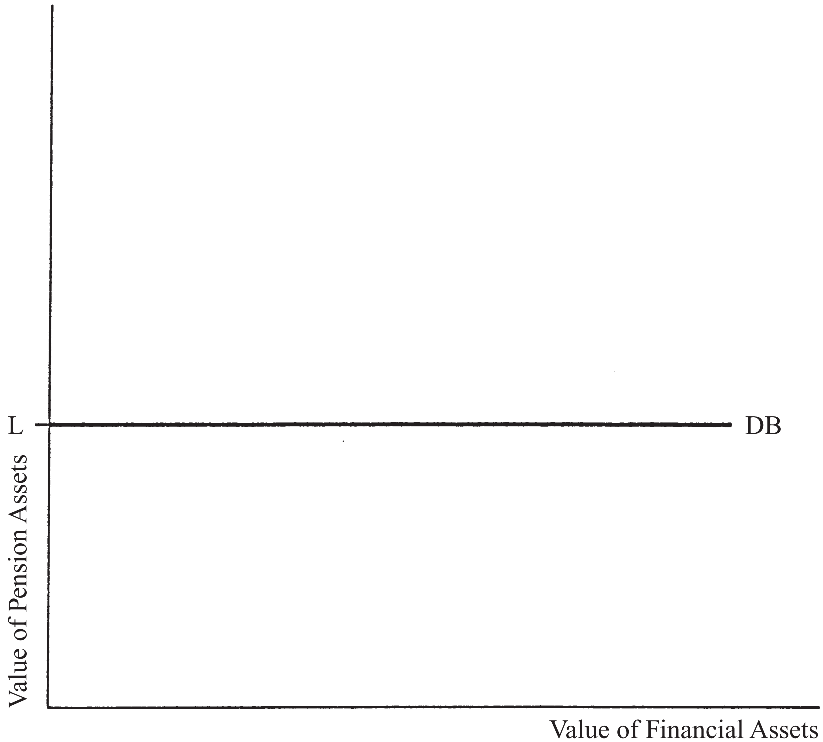


*Figure 2: A Defined Benefit Pension Scheme*

(almost) certain to be exercised. If the value of the assets is less than the exercise price, so that the scheme is showing an actuarial deficit, the member will exercise his or her put option against the sponsor who will then be required to make a deficiency payment $(L - A)$. If, on the other hand, the value of the assets exceeds the exercise price, so that the scheme is showing an actuarial surplus, the sponsor will exercise his or her call option against the member and recover the surplus $(A - L)$.

It is clear from this how DB and DC schemes are related. A DC scheme is invested only in the underlying assets. A DB scheme is invested in a portfolio containing the underlying assets (and so is, in part, a DC scheme) *plus* a put option *minus* a call option on these assets[3]:

$$\begin{aligned} DB &= L \\ &= A + P - C \\ &= DC + P - C. \end{aligned} \tag{1}$$

[3] See Light and Perold (1987), note 2, for a different application of this equation based on put options issued by the U.S. Pension Benefit Guaranty Corporation.

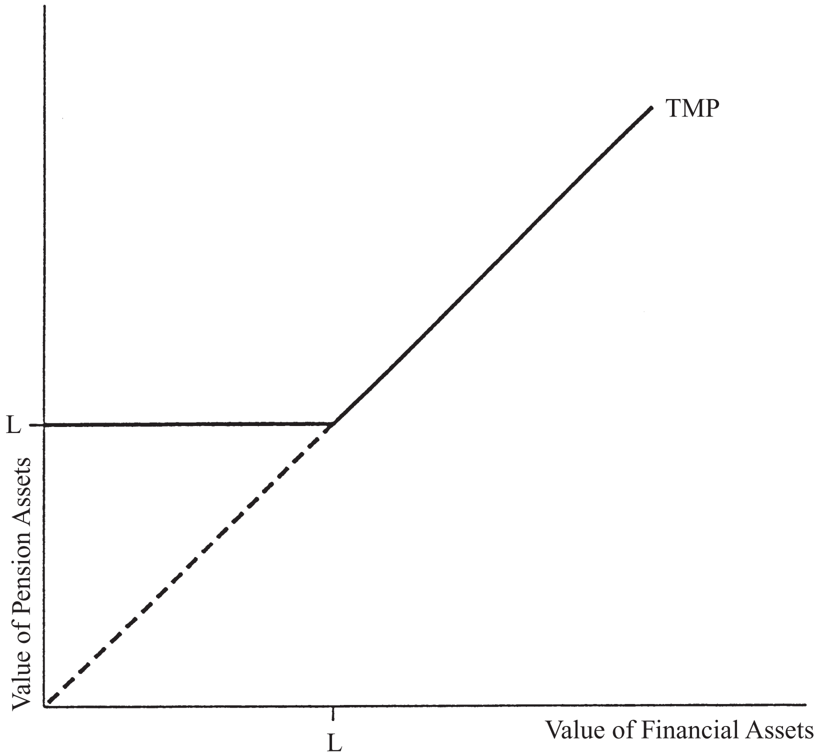


*Figure 3: A Targeted Money Purchase Pension Scheme*

Figure 5 shows that the TMP pension can be replicated using a long (protective) put option ($P$) on the underlying assets of the fund ($A$) with an exercise price ($L$). The put option is held by the scheme member and written by the scheme sponsor. On the retirement date of the member, which again coincides with the expiry date of the option, the option will be exercised if the value of the assets is less than the exercise price. The effect of the option is to place a floor on the value of the pension received by the member. The present value of the TMP pension is the larger of the two present values provided by the DC and DB schemes, whatever the value of the underlying assets:[4]

$$\begin{aligned} TMP &= A + P \\ &= \text{Max}(A,\ L) \\ &= \text{Max}(DC,\ DB) \\ &= C. \end{aligned} \tag{2}$$

[4] We assume here that the target pension with the TMP scheme is the same as that with the DB scheme, but this need not be the case.

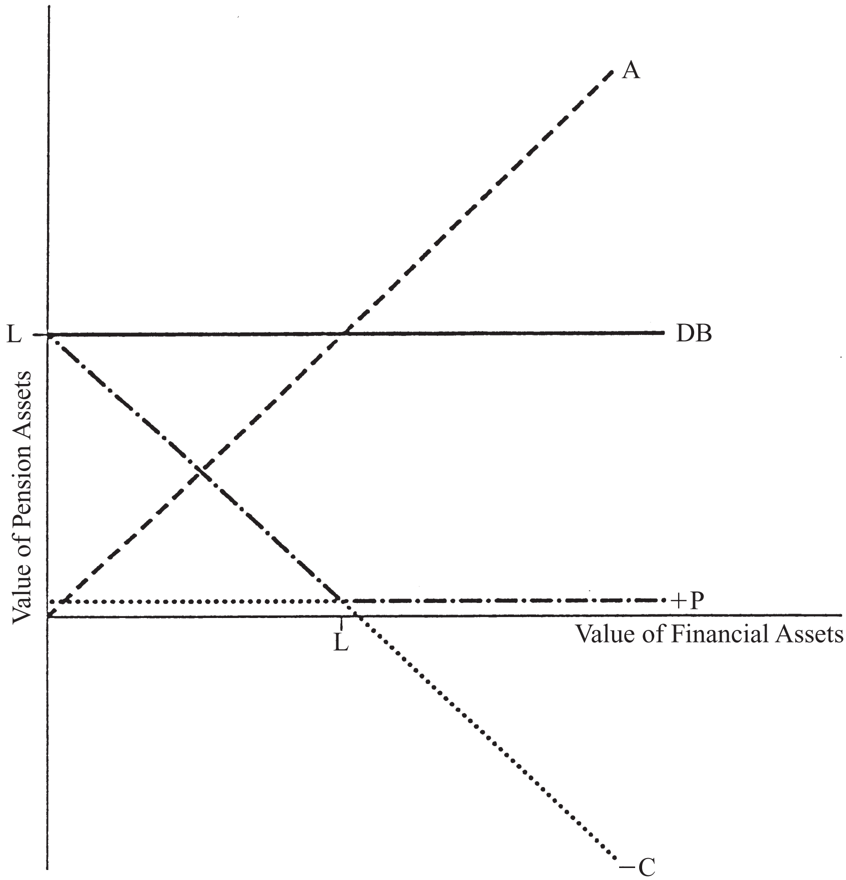


*Figure 4: The Option Composition of a Defined Benefit Scheme*

This implies that a TMP scheme is equivalent to a call option (or floor) held by the member on the underlying pension assets with an exercise price $L$.[5] The call option will only be exercised if, on maturity, $A$ exceeds $L$.

Life assurance policies also have option features attached to them. Smith (1982), for example, demonstrates that a whole life policy is composed of the following set of options: an option to renew the policy at guaranteed renewal premiums, optional modes of receiving payment in the event of a death claim, optional methods by which dividends can be received (for a participating policy), a policy loan option and a surrender option (i.e., a policy lapse).

5 The TMP scheme is therefore also equivalent to an endowment insurance scheme, see, e.g., Gemmill (1993), chapter 10.3.

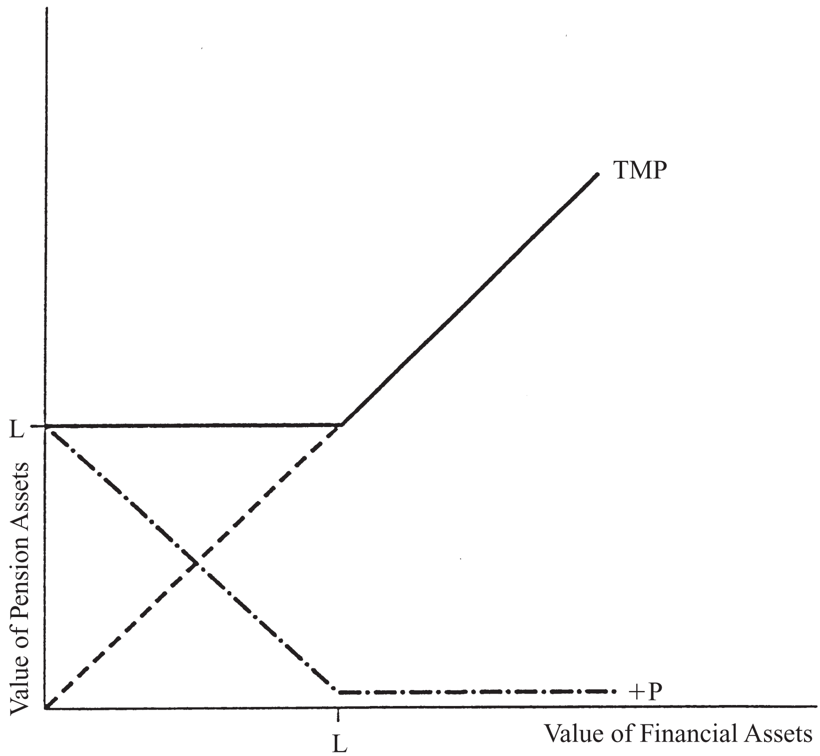


*Figure 5: The Option Composition of a Targeted Money Purchase Pension Scheme*

Smith examines the policy loan option. This option enables the policyholder to borrow almost the entire cash surrender value at the crediting rate. An increase in market interest rates above the crediting rate may induce policy holders to exercise the loan option, while a fall in market interest rates below the crediting rate will lead to funds being returned. If a rise in interest rates causes the loan option to be exercised, this will have two effects on the life company's position. First, it will lower the marginal return on the life company's assets as a high-yielding asset is liquidated to provide the funds for the lower-yielding policy loan. Second, unless the life company has sufficient liquid, capital-certain assets, it will have to sell fixed-income bonds at a loss to meet the demand for policy loans. When interest rates subsequently fall back and the policy loan funds are returned, the opposite set of effects will occur, in particular, the bonds will be re-purchased at much higher prices. It is clear from this analysis that a policy loan clause is equivalent to the following sequence of options held by the policyholder: a put option which if it exercised is replaced by a call option. The put gives the policyholder the right to sell the policy for its cash value when interest rates rise above the exercise price (crediting rate), while the call gives the policyholder the right to buy back the policy when interest rates fall below the exercise price. The options are American since they can be exercised before the maturity of the policy.

Figure 6 shows the position from the viewpoint of the policyholder. The policyholder has a long put option (*P*) on the financial assets (typically bonds) underlying the policy. The put will be exercised when the market value of the bonds falls below the cash value of the policy (*L*). Market interest rates will be high at this time and the proceeds will be invested in high-yielding, low-price bonds. If market interest rates subsequently fall back, the value of the bonds will rise above the cash value of the policy, and the policyholder will return the borrowed funds to the life company and reinstate the policy. This is equivalent to the policyholder having a long call option (*C*) on the bonds at an exercise price equal to the cash value of the the policy. This option will be exercised when it is in-the-money and, in effect, the life company has to buy back high-priced bonds from the policyholder to enable the policy holder to reinstate the policy. The wealth profile from the policyholder's viewpoint is identical to that of a member of a TMP pension scheme (see Figure 3 and Note 5).

Albizzati and Geman (1994) examine the surrender option. It is optimal for a policyholder to surrender a contract if the rate of return earned after surrender more than compensates for the loss of the policy's options. This is demonstrated in Figure 7 which shows the case where it is optimal for the policyholder to surrender his or her existing policy at the end of year 3: even after upfront management fees are taken into account, the surrender value of the new policy exceeds the surrender value of the existing policy by the time the existing policy matures in year T.

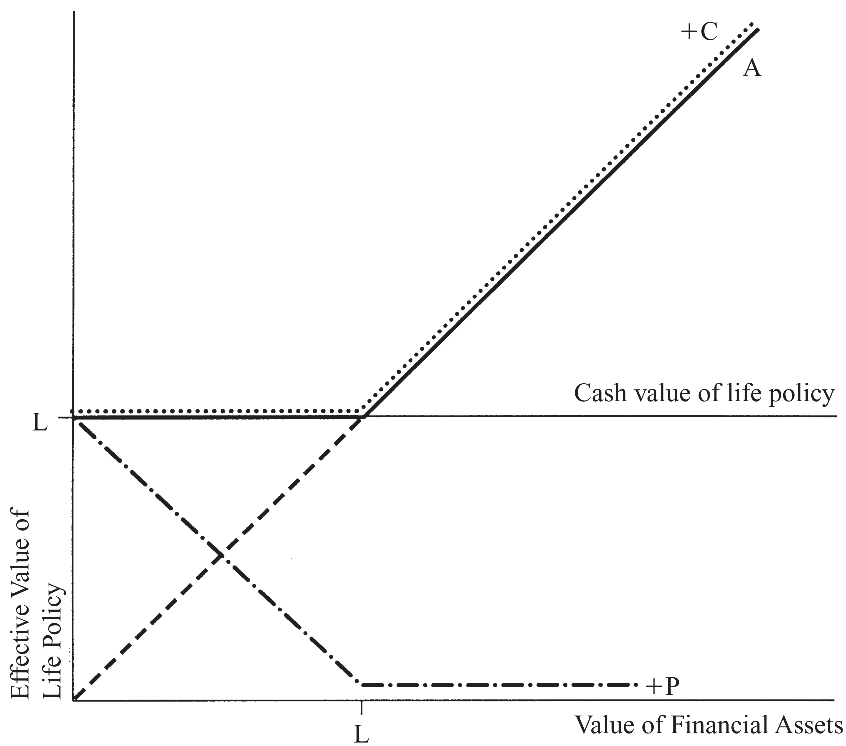


*Figure 6: The Policy Loan Option*

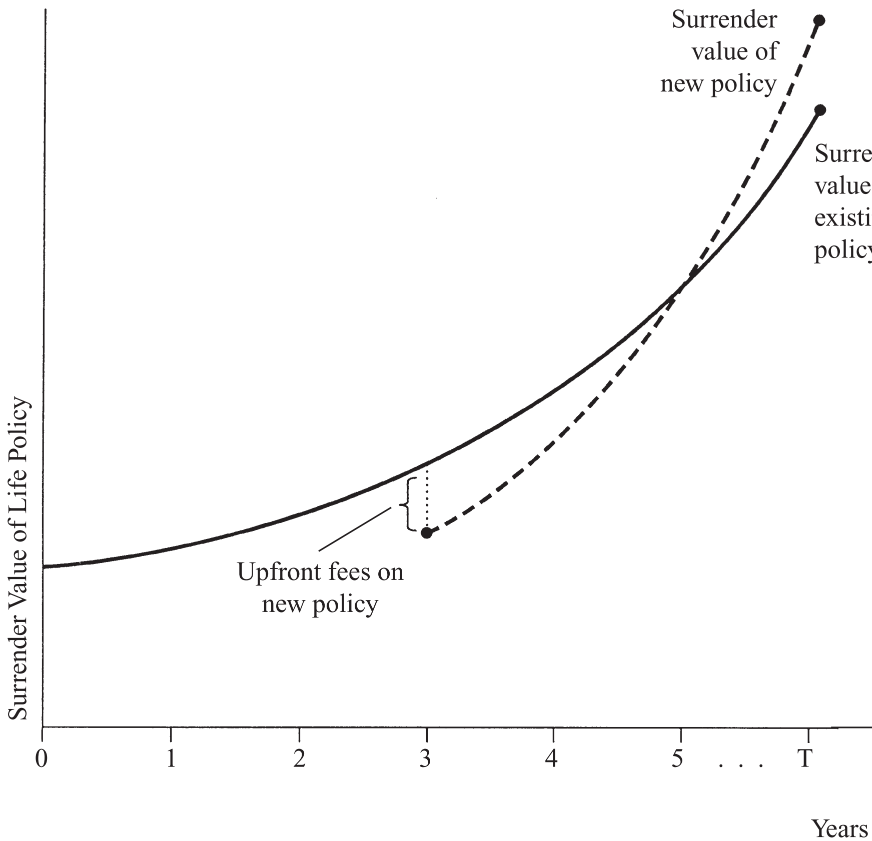


*Figure 7: The Surrender Option*

## 4. Valuing the options

We can now examine in more detail how the options are valued, beginning again with the pension schemes.

The expected value of the member's pension assets at retirement depends on the member's starting income ($Y$), the contribution rate into the scheme ($\gamma$, e.g., 12.2 per cent of income), the expected growth rate in income ($g_Y$), the expected yield on the financial assets purchased with the contributions ($r_F$), the rate of tax relief on contributions ($\tau$),[6] and the number of years of pensionable service ($T$). Assuming that the appropriate discount rate used to discount the remaining contributions equals the expected yield on financial assets, the present value of a member's pension assets at any date $t$ will equal the value of the

[6] We assume for simplicity that the tax rates are the same for the member and the sponsor.

accumulated financial assets ($F_t$) *plus* the discounted value of the remaining contributions ($X_t$), both of which are functions of the above variables:[7,8]

$$A_t = F_t(\gamma, Y, g_Y, \tau, r'_{F1}, \ldots, r'_{Ft}) + X_t(\gamma, Y, g_Y, \tau, r_F, T), \; t = 1, T, \tag{3}$$

where $r'_{F1}, \ldots, r'_{Ft}$ are the realized yields on financial assets in years $1, \ldots, t$.

With a DB scheme, the liabilities at retirement ($T$) depend on the pension at retirement ($Z$), the expected growth rate in the pension ($g_Z$) and the actuarial length of retirement ($M$). Suppose that the retirement pension, $Z$, is equal to some proportion ($\theta$, e.g., two-thirds) of the expected retirement salary ($Y(1+g_Y)^{T-1}$). Therefore, the present value of the liabilities at $t$ is a function of these variables (assuming a discount rate of $r_L$):[9]

$$L_t = L_t(Z, g_Z, M, r_L, T), \qquad t = 1, T. \tag{4}$$

The surplus with a DB scheme is given by (using (3) and (4)):

$$S_t = A_t - L_t, \qquad t = 1, T. \tag{5}$$

With a DC scheme, equation (3) equals the present value of both the assets and liabilities, so that there is no surplus. With a TMP scheme, the liabilities are the larger of equations (3) and (4), but, as with a DC scheme, there is no surplus.

The options embodied in the DB and TMP schemes have the following characteristics. They are European options, since they cannot be exercised before the retirement date.[10] In

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[7] This paper uses the *implicit lifetime contract method* or *prospective benefit funding method* of determining pension liabilities and assets (see, e.g., Disney and Whitehouse (1996), or Haberman and Sung (1994)). This method assumes that the member will work until normal retirement age and then draw a pension until death. This contrasts with the *accrued benefit funding method* which determines pension liabilities and assets only up to the date of accrual and disregards likely future service (see, e.g., Institute and Faculty of Actuaries, 1984).

[8] The precise formula for calculating pension assets is (assuming cash flows arise at the end of the period):

$$A_t = F_t + X_t$$
$$= \sum_{k=1}^{t} \frac{\gamma Y(1+g_Y)^{k-1}}{(1-\tau)} \prod_{j=k+1}^{t} (1 + r'_{Fj}) + \sum_{k=t+1}^{T} \frac{\gamma Y}{(1-\tau)} \left(\frac{1+g_Y}{1+r_F}\right)^{k-1},$$

where $r'_{Fj}$ is the realized return on investments in the fund in year $j$ and $r_F$ is the expected return on financial assets in the future. This is consistent with conventional practice in the actuarial profession and parts of the economics profession (e.g., Tepper, 1981). However, other economists suggest using the after-tax rate of return on corporate bonds, e.g., Copeland (1984). This is more in line with practice with the accountancy profession which recommends using the yield on long-term government bonds (see, e.g., Federal Accounting Standard 87 and International Accounting Standard 19).

[9] The precise formula for calculating liabilities is:

$$L_t = \sum_{k=1}^{M} Z\left(\frac{1+g_Z}{1+r_{L1}}\right)^k \left(\frac{1}{1+r_{L2}}\right)^{T-t}.$$

Conventional actuarial practice assumes that the discount rate from the retirement date onwards ($r_{L1}$) is the same as the discount rate on $M$-year government bonds (on the grounds that such bonds are used to finance annuities). For the discount rate from the retirement date back to date $t$ ($r_{L2}$), some suggest using the same rate as that for discounting projected contributions (i.e., $r_F$), while others suggest using a long-term government bond yield, since this reflects more accurately the volatility attached to the future annuity stream.

[10] More sophisticated versions of the model could contain options which allow for earlier termination of the pension scheme on the grounds of redundancy (exercised by the sponsor) or ill-health (exercised by the member) etc. These are American options of the kind that are also relevant to life companies and which are discussed later in this section.

addition, the underlying asset does not make payouts prior to the expiry date of the option. However, the most important feature of the options is that the exercise price is not constant, as in the standard Black–Scholes (1973) model, but is equal to the value of the liabilities. The appropriate option valuation model is based on a modification to the Black–Scholes model which recognizes that the options in equations (1) or (2) are options to exchange risky assets at an exercise price that is indexed to the uncertain value of the liabilities (see Fischer, 1978, and Margrabe, 1978).

The value of the call option in equation (1) is given by:

$$C_t = N(d1_t)A_t - N(d2_t)L_t, \tag{6}$$

where:

$$d1_t = \frac{\ln(A_t/L_t) + 0.5\sigma_{St}^2(T-t)}{\sigma_{St}\sqrt{T-t}} \tag{7}$$

$$d2_t = d1_t - \sigma_{St}\sqrt{T-t} \tag{8}$$

$$\sigma_{St}^2 = \sigma_{At}^2 + \sigma_{Lt}^2 - 2\sigma_{Alt}, \qquad t = 1, T. \tag{9}$$

$N(d1_t)$ and $N(d2_t)$ are normal distribution functions (lying in the range [0, 1]) evaluated at $d1_t$ and $d2_t$ respectively. Equation (9) is the variance ($\sigma_{St}^2$) of the surplus (5) (i.e., the (square of) *surplus risk*) which depends on the standard deviations of the rates of return on assets ($\sigma_{At}$) and liabilities ($\sigma_{Lt}$), and the covariance between the returns on assets and liabilities ($\sigma_{ALt}$).[11]

From equation (3), we can see that the volatility of the growth rate in pension assets depends on: (i) their duration (see Macaulay, 1938), which equals the weighted average of the durations of the accumulated financial assets ($D_{Ft}$) and of the stream of remaining contributions ($D_{Xt}$):[12]

$$D_{At} = \alpha_t D_{Ft} + (1-\alpha_t)D_{Xt}, \qquad t = 1, T. \tag{10}$$

(where $\alpha_t$ is the share of the existing financial assets in total pension assets at time *t*, i.e. $\alpha_t = F_t/(F_t + X_t)$), and (ii) the standard deviations of the the rates of change in the yields on financial assets ($\sigma_r$) and in the growth rate in earnings ($\sigma_g$). Duration measures the sensitivity (or elasticity) of the present value of a stream of future cash flows to a change in interest rates (see Blake, 1990, chapter 5). As a first-order approximation, the variance of the assets is given by:[13]

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[11] Note that the option valuation model uses the standard deviations of the *rates of change* rather than the *levels* of these variables.

[12] Duration is the weighted average maturity of a stream of future cash flows (using relative discounted cash flows as weights). For example, in the case of the stream of contribution payments:

$$D_{Xt} = \sum_{k=t+1}^{T} \left( (k-t)\frac{\gamma Y}{(1-\tau)X_t}\left(\frac{1+g_Y}{1+r_F}\right)^{k-1} \right).$$

[13] This equation is derived in Blake (1998, note 10). We assume for simplicity that r and g are uncorrelated; that there are parallel yield curve shifts so that the variance of the interest rate is constant at all maturities; and that the standard deviations of earnings growth ($g_Y$), pensions growth ($g_Z$), and, later, inflation growth ($g_I$) and dividend growth ($g_E$) are all equal.

$$\sigma^2_{At} = D^2_{At}(\sigma^2_r + \sigma^2_g) + \eta^2_{\mathrm{A}}, \qquad t = 1, T, \tag{11}$$

where $\eta_{\mathrm{A}}$ is the specific risk on the assets.

In a similar way, the volatility of the growth rate in liabilities depends, from equation (4), on their duration (see, e.g., Langetieg *et al.*, 1986):[14]

$$D_{Lt} = D_{LT} + (T - t), \qquad t = 1, T, \tag{12}$$

where $D_{LT}$ is the duration of the liabilities as of the retirement date. As a first-order approximation, the variance of the liabilities is given by:

$$\sigma^2_{Lt} = D^2_{Lt}(\sigma^2_r + \sigma^2_g) + \eta^2_{\mathrm{L}}, \qquad t = 1, T, \tag{13}$$

where the standard deviation of the growth rate in the pension is $\sigma_g$ and $\eta_L$ is the specific risk on the liabilities. The covariance between assets and liabilities is given by:

$$\begin{aligned}\sigma_{ALt} &= \rho_{AL}\sigma_{At}\sigma_{Lt} \\ &= D_{At}D_{Lt}(\sigma^2_r + \sigma^2_g) + \eta_{AL}, \qquad t = 1, T,\end{aligned} \tag{14}$$

where $\eta_{AL}$ is the covariance between the specific risks on assets and liabilities.

The value of the put options in equations (1) and (2) is derived from *put-call parity* as (using (5)):

$$P_t = C_t + L_t - A_t = C_t - S_t, \qquad t = 1, T. \tag{15}$$

Two important features of equations (6) and (15) are that the option values do not depend explicitly on the riskless rate of interest as in the standard Black–Scholes model, and that the appropriate definition of risk is not the risk, given by (11), attached to the *pension assets* (3), but the risk, given by (9), attached to the *pension surplus* (5). Both these features follow because the pension liabilities provide a natural hedge for the pension assets against both interest rate and inflation risk.

The rationale for the first feature comes from the Black–Scholes innovation of constructing a riskless hedge portfolio. In order to do this, it is necessary to hedge against changes in both the value of the underlying assets and the exercise price. Changes in asset values are hedged by holding the assets. The cost of this hedge is equal to the rate of return on the assets. But because the assets themselves are held in the portfolio, the return from the portfolio exactly offsets the cost of the hedge against changes in asset values. Therefore, the rate of return on assets does not appear in the option pricing formula. Because the hedge portfolio is riskless and generates the riskless rate of return, only the riskless rate of interest appears in the standard Black–Scholes formula. Changes in the exercise price are hedged by holding in the portfolio assets whose returns are perfectly correlated with changes in the exercise price, i.e., with changes in the value of the liabilities. This is achieved by holding, as part of the main portfolio, a *liability immunizing portfolio* ("LIP"), i.e., a portfolio that exactly tracks any changes in the value of the liabilities. While the LIP will be a risky component of the total portfolio, it will be riskless relative to the liabilities that it is

[14] The precise formula for the duration of liabilities is:

$$D_{Lt} = \sum_{k=1}^{M} \frac{kZ}{L_T}\left(\frac{1 + g_Z}{1 + r_{L1}}\right)^k + (T - t).$$

immunizing and so it too will generate the riskless rate of return. Therefore, the rate of return on the hedge portfolio in this case is zero, since the return on the liability hedge exactly offsets the return on the asset hedge.[15]

The rationale for the second feature comes from the fact that pension asset and liability values respond to shocks in a similar way. From equations (3) and (4) it is clear that the main sources of shocks are unexpected changes in interest rates and growth rates (in earnings or pensions, say). For example, an unexpected increase in interest rates reduces the present values of both assets and liabilities, while an unexpected increase in growth rates has the opposite effect. The volatilities of interest rates and growth rates are common to both assets and liabilities. The differential effects of these volatilities on the variances of assets and liabilities comes from the differing durations of assets and liabilities (see (10) and (12)), as seen in equations (11), (13) and (14). Substituting these into equation (9) we get:

$$\sigma_{St}^2 = (D_{At} - D_{Lt})^2(\sigma_r^2 + \sigma_g^2) + \eta_A^2 + \eta_L^2 - 2\eta_{AL}, \qquad t = 1, T. \tag{16}$$

This shows that the volatility of the surplus depends on the squared *duration gap* between assets and liabilities, the variances of the rates of change in yields and growth rates, and the relationship between the specific risks on assets and liabilities. If the asset portfolio is constructed to have returns that are perfectly correlated with changes in the value of the liabilities, then $\eta_A^2 = \eta_L^2 = \eta_{AL}$, and the terms involving $\eta$ in (16) vanish. If, in addition, the duration of the assets is kept equal to that of the liabilities, then surplus risk can be eliminated altogether.

The options embodied in life assurance products are somewhat more complicated than those involved in the pension schemes that we have just considered. We can demonstrate this using the surrender option which has been valued by Albizatti and Geman (1994). The Fischer–Margrabe model cannot be used directly to value surrender options for two reasons. First, the Fisher–Margrabe model, like the original Black–Scholes model, assumes a fixed interest rate, whereas the surrender option arises only in the context of a stochastic interest rate environment. Second, the Fischer–Margrabe model, again like the original Black–Scholes model, is for a European option, whereas the surrender option is American and can be exercised before the contract renewal date. Albizatti and Geman resolve these problems by recognizing that insurance companies operate a portfolio of insurance policies (and hence benefit from a risk-pooling or averaging effect) and also typically allow surrenders to take place only at the end of the calendar year. They also make the simplifying assumptions that, while interest rates are stochastic, their covariance structure is deterministic and that surrenders in a particular year are independent of surrenders in previous years.

These features permit the surrender option to be valued by arbitrage as the expectation of random cash flows arising at specified future dates and discounted using stochastic interest rates. In other words, the surrender option on a $T$-year life policy can be valued as the sum (between $t = 1, T - 1$) of a series of European Fischer–Margrabe put options of the form:

$$P_t = \mathrm{e}^{t\xi rs(0,T)} B_t N(d1_t) - N(d2_t), \tag{17}$$

where $rs(0, T)$ is the $T$-year spot yield at $t = 0$, $B_t$ is the value at time $t$ of a zero-coupon bond

[15] See also Leibowitz (1986b) who argues that liabilities can be treated as short positions in assets and liability returns can be treated in a commensurate way.

paying the maturity value of the policy at time $T$, and $\xi$ is a fraction less than one (e.g., 0.9), reflecting the penalty in terms of a lower effective return from early surrender. Also in (17):

$$d1_t = \frac{(rf(0,\ t,\ T) - \chi_t) + 0.5\sigma^2_{rs,t,T}(T - t)}{\sigma_{rs,t,T}} \tag{18}$$

and

$$d2_t = d1_t - \frac{cov(rs(t,\ T - t),\ rs(t,\ T))}{\sigma_{rs,t,T}}(T - t), \tag{19}$$

where

$$\chi_t = \frac{rs(0,\ T)T - \ln(K_t)/\xi}{T - t}, \tag{20}$$

where $rf(0,\ t,\ T)$ is the $T$-year forward rate from time $t$, observed at time 0, $\sigma^2_{rs,t,T}$ is the (deterministic) variance of the $T$-year spot yield curve at time $t$, $cov(rs(t,\ T - t),\ rs(t,\ T))$ is the (deterministic) covariance between the $(T - t)$-year and $T$-year spot yields at time $t$, $K_t$ is the (net of upfront management fees) value at time $t$ of a new $(T - t)$-year policy replacing the one the policyholder surrenders, and where $N(d1_t)$ and $N(d2_t)$ are normal distribution functions evaluated at $d1_t$ and $d2_t$ respectively. At the end of each year, the policyholder has the choice of keeping the existing policy or exchanging it for a new policy on better terms. The life company, aware of the possibility of surrender on an annual basis and also of the proportion of the original set of policies still alive at each surrender date, will invest the premiums on the policy in securities with predictable liquidation values, typically bonds (in the Albizzati–Geman framework, they are zero-coupon bonds).

## 5. The optimal management of pension fund and life company assets

Having considered the composition and valuation of the options embodied in pension schemes and life policies, we are now in a position to examine the consequences of this for the optimal management of both pension fund and life company assets, again beginning with pension funds.

We will assume that the objective of the sponsors of a DB scheme is to manage the pension assets over time so as to minimize the surplus risk (16) on a year-by-year basis, subject to ensuring that the surplus (5) never falls below zero. This is the rationale underlying *asset-liability management* ("ALM"), otherwise known as *surplus (*or *shortfall* or *solvency) risk management and insurance* (see, e.g., Leibowitz, 1986c, Leibowitz and Henriksson, 1987, Kritzman, 1988, Bodie, 1991, and, for recent overviews, van der Meer and Smink, 1993, Smink and Van der Meer, 1997);[16] Blake (1992) reports a survey by Greenwich Associates

[16] The optimal management of the pension fund assets involves the solution to a dynamic programming problem which, because of standard assumptions (namely, we will assume that the state variables are generated by time-independent Markov processes and that the surplus risk function is time-separable) reduces to a sequence of single-period optimization problems. These single-period problems have sequential solutions: first, ensure there are sufficient assets to meet liabilities; and second, choose the asset allocation to minimize surplus risk, conditional on the properties of the individual asset categories. Haberman and Sung (1994) use a similar dynamic programming framework to derive the optimal time path of contributions into a DB scheme which minimizes both contribution rate risk and solvency risk.

which shows that 30 per cent of U.S. pension funds were using ALM by 1990. In contrast, the objective of the sponsors of a TMP scheme is simply to minimize the risk of generating a shortfall below the target value for the pension fund assets, while the sponsors of a DC scheme are not concerned with either the surplus or with surplus risk, but instead can choose the asset structure over time that maximizes the member's tradeoff between reward and risk, given his or her degree of relative risk aversion. We will suppose that a typical DC scheme member's reward–risk preferences can be represented by a risk-adjusted utility function of the form (where $\beta$ is a constant relative risk aversion parameter $\beta$, see Merton, 1969):

$$\begin{aligned} U &= \text{Expected return} - \text{Risk penalty} \\ &= r_F - \tfrac{1}{2}\beta\sigma_A^2. \end{aligned} \tag{21}$$

The greater the asset risk assumed, the greater the expected return but also the greater the risk penalty; similarly the greater the degree of risk aversion, the greater the risk penalty for a given degree of asset risk. The fund manager's objective is to choose the asset allocation that maximizes (21).

There are two principal techniques for ALM: *immunization* (Redington, 1952) and *portfolio insurance* (Leland, 1980, Gatto *et al.*, 1980, Brennan and Solanki, 1981, Leland and Rubinstein, 1981). The purpose of classical immunization is to generate an assured return on the pension assets over the investment horizon. This is achieved by eliminating surplus risk, which, from equation (16), requires restructuring the pension assets to have both the same duration as the liabilities and returns that are perfectly correlated with changes in the value of the liabilities, i.e., investing in a LIP. With classical portfolio insurance, in contrast, the sponsor seeks to lay off the downside risk that he faces from the exercise of the put that was sold to the member through the creation and management of his own protective put option, while preserving the upside potential of the asset portfolio. However, the scheme member faces the opposite risk and may wish to protect his downside risk through the creation and management of a protective call option. We can therefore conceive of a portfolio insurance strategy (which we call *bi-directional portfolio insurance*) that attempts to eliminate the downside risks of both the sponsor and the member, but this will be at the cost of eliminating the upside potential in both cases. However, the fund management strategy needed to achieve bi-directional portfolio insurance will be identical to that required for classical immunization as we now show.

Classical immunization requires the surplus risk to be zero. However, when the surplus risk is zero, the values of the calls and puts are related solely to the size of the surplus (see equations (6) to (9) and (15)). So if the pension scheme is being fully funded on a year-by-year basis and the asset portfolio is being continuously immunized to liabilities (so that the surplus is zero), the puts and calls both have zero value since neither will be exercised. The pension scheme's fund manager can therefore replicate the payoff pattern of the put and call options (i.e., implement bi-directional portfolio insurance) by ensuring that the liabilities are immunized continuously over time using assets invested in a LIP.

However, as we show shortly, it will be impossible to invest in a LIP for the whole investment horizon, so the optimal asset allocation strategy will need to use a mixture of financial assets: equities, index bonds and conventional fixed-interest bonds. We will assume that the realized returns on these assets are determined by the following variation on the standard linear market model in finance:

$$r'_{Et} = r_f + \pi D_E + \epsilon_{Et}$$

$$r'_{It} = [\rho_f(1+g_I) + g_I] + \pi D_{It} + \epsilon_{It}$$
$$r'_{Bt} = r_f + \pi D_{Bt} + \epsilon_{Bt}, \qquad t = 1, T, \tag{22}$$

where $\pi$ is the (stochastic) *duration risk premium*, $D_E$, $D_{It}$ and $D_{Bt}$ are the durations of, and $\epsilon_{Et}$, $\epsilon_{It}$ and $\epsilon_{Bt}$ are white noise components of, the returns on equities, index bonds and conventional bonds, respectively, and where $\rho_f$ is the (constant) real return on index bonds and $g_I$ is the expected inflation rate; expected returns are denoted $r_E$, $r_{It}$, and $r_{Bt}$. We further assume that the specific components of the returns on assets, $\epsilon_{Et}, \epsilon_{It}$ and $\epsilon_{Bt}$, have the following properties: they have zero mean, constant variance, and are serially and contemporaneously uncorrelated with each other, but that $\epsilon_{It}$ is perfectly correlated with the rate of change of the liabilities, so that $\eta_I^2 = \eta_L^2 = \eta_{IL}$. This is because index bonds (whose values are perfectly correlated with changes in the price level) are assumed to be perfectly correlated with pension liabilities (whose values are perfectly correlated with changes in the wage level), because changes in wages and prices are, in turn, assumed to be perfectly correlated. Comparing (22) with the standard market model, $\pi$ corresponds with the market risk premium (and, like the market risk premium, is assumed to be identical across securities) and duration corresponds with beta.

The motivation for this model again lies with the desire to utilize common sources of variability between assets and liabilities. In the previous section we argued that the volatility of interest rates and growth rates would be key common sources of volatility. We will therefore assume that the variances of the returns on equities, index bonds and conventional bonds, and the covariances between them can be modelled as follows:[17]

$$\sigma_E^2 = D_E^2(\sigma_r^2 + \sigma_g^2) + \eta_E^2, \qquad \sigma_{It}^2 = D_{It}^2(\sigma_r^2 + \sigma_g^2) + \eta_I^2, \qquad \sigma_{Bt}^2 = D_{Bt}^2\sigma_r^2 + \eta_B^2,$$
$$\sigma_{EIt} = D_E D_{It}(\sigma_r^2 + \sigma_g^2), \qquad \sigma_{EBt} = D_E D_{Bt}\sigma_r^2, \qquad \sigma_{IBt} = D_{It} D_{Bt}\sigma_r^2, \tag{23}$$

where $\eta_E, \eta_I$, and $\eta_B$ are the specific risk components of the returns on equities, index bonds, and conventional bonds. From (22) and (23), it is clear that the variance of $\pi$ is given by $(\sigma_r^2 + \sigma_g^2)$: the variance of the market portfolio in the market model is replaced by the sum of the variances of its components in our model. Notice that the variance of fixed-income bond values depends only on the variance of interest rates.

We will assume for simplicity that the duration of equities is constant and given by:

$$D_E = \frac{1 + r_E}{r_E - g_E}, \tag{24}$$

which holds under the assumption that $g_E$, the expected growth rate in dividends, is constant.[18] But we will assume that index and conventional bonds with different durations are available for inclusion in the optimal portfolio.

The optimal portfolio of financial assets chosen by the fund manager will be determined in a sequence of *three* stages over the lifetime of the scheme, with the length

[17] We assume for simplicity that the sources of systematic risk in financial assets (i.e., interest rates and growth rates) are the same as that for the underlying pension assets and liabilities themselves.

[18] See, e.g., Boquist *et al.* (1975). As calculated using this formula, the duration of equities will be substantially higher than that for most bonds: it assumes that $g_E$ does not respond to changes in $r_E$. Not all investigators believe that equity duration is as great as implied by this formula, e.g. Leibowitz (1986a).

of each stage depending on the relationship between the durations of the pension liabilities and pension assets (Samuelson, 1989, has called this investment strategy *age-phasing*; it is better known as *lifestyle* fund management). To begin with, the duration of the pension liabilities will greatly exceed that of the pension assets (financial assets *plus* remaining contributions). This is because the duration of the remaining contributions is lower than that of the liabilities and because initially the weight of financial assets in total pension assets will be negligible and also there do not exist financial assets with sufficiently high durations to compensate for this. The objective at the first stage therefore is to build up the duration of the financial assets as quickly as possible. This involves investing the entire portfolio of financial assets in the highest duration (and therefore in the highest risk) assets of all, i.e., in equities. The first stage will last until the duration of the assets has been raised to equal that of the liabilities.

The objective at the second stage is to build up the investment in the liability immunizing portfolio, while preserving the equality between the durations of the pension assets and pension liabilities. The LIP is selected to be perfectly correlated with and to have precisely the same duration as the liabilities. The LIP will contain index bonds that are perfectly correlated with and have the same duration as the liabilities. It will generally not be possible at the second stage to switch immediately and completely into index bonds, since the duration of index bonds is likely to be less than that of equities; so again the investment in index bonds will have to be built up gradually in a way that preserves the equality of duration between assets and liabilities.

Thus far we have accumulated financial assets in such a way that there is both duration-matching and perfect correlation between pension assets and liabilities. The first two stages involve investing in the riskiest assets in order to build up duration, regardless of the degree of risk aversion of the sponsor or member. This is counter-intuitive: we invest in the *riskiest* financial *assets* in order to *reduce the risk* to pension *liabilities*. At the same time, we are taking advantage of the *equity risk premium* ($\pi D_E$).[19] At the third stage, the objective is to continuously immunize liabilities using the LIP[20] and simultaneously to achieve the desired reward–risk configuration on any *free assets* in excess of the LIP. It is only at this stage that the fund manager is able to take into account the degree of risk aversion of the sponsor or the member, depending on the type of scheme. The lower the degree of risk aversion, the greater the degree of risk and hence expected return that can be taken on by the fund manager. With a DB scheme, it will be the sponsor's degree of risk aversion that is relevant, while with DC and TMP schemes, the optimal portfolio of financial assets will depend on the member's attitude to risk. A DC scheme operates as a stage 3 scheme throughout its entire life. With a TMP scheme, all the downside risk is borne by the sponsor, while all the upside potential is retained by the member. The member, in effect, has a floor on the value of the pension assets, paid for with a put bought from the sponsor. The sponsor, in turn, hedges against the exercise of this

[19] There is no five-year period in the history of any advanced financial system over which equities have not outperformed bonds. Further, Samuelson (1989, 1991, 1992) argues that, when security returns are mean-reverting, it is rational for long-horizon investors such as pension funds to invest more heavily in high-risk equities than in low-risk bonds during the early years of a pension scheme and then to switch into bonds as the horizon shortens. Only when security returns are pure random walks is it the case that the optimal asset allocation does not depend on the length of the investment horizon (see, e.g., Samuelson, 1963 and Merton and Samuelson, 1974).

[20] The value of the LIP always equals the present value of liabilities *minus* the present value of remaining contributions.

put by switching out of risky securities into cash as the risky securities fall in value below the floor level.

Once the duration of the financial assets has been built up sufficiently to equal (taking into account the duration of the remaining contributions) the duration of the pension liabilities (so that surplus risk is zero), a highly risk averse scheme member or sponsor will be satisfied with the LIP as the only portfolio of financial assets needed. However, a less risk averse member or sponsor might wish to take on some additional *asset risk* by investing in a portfolio of risky assets, comprising equities, index bonds and conventional bonds; this is equivalent to taking on some additional diversifiable risk in an otherwise well-diversified portfolio. The optimal weights in equities, index and conventional bonds in the portfolio can be found from a standard source such as Elton and Gruber (1995, chapter 6). The expected return on financial assets ($r_F$) is calculated as the weighted average of the expected returns on the individual asset classes, derived, using these optimal weights, from equation (22).

The dynamic asset allocation strategy outlined here can be compared with the dynamic reallocations of the asset portfolio between equities and cash proposed by Leland and Rubinstein (1981), and is as simple to implement as, say, the constant proportion portfolio insurance strategy of Perold and Sharpe (1988). For an earlier approach to pension fund immunization, see Keintz and Stickney (1980).

ALM is also appropiate for life companies in cases where they can determine the duration of their liabilities. Stowe and Watson (1985) provide empirical evidence and Lamm-Tennant (1989) provides survey evidence on U.S. life companies' use of ALM. Lamm-Tennant's survey shows that three-quarters of U.S. life companies used duration-based ALM techniques of the kind discussed above in the case of pension funds. Stowe and Watson's empirical evidence indicates that in the case of *separate accounts* (e.g., the accounts of pension schemes managed on a segregated basis) the assets and liabilities appear to be almost perfectly duration-matched. Since life companies tend to have a higher proportion of their liabilities fixed in nominal terms compared with pension funds, they will hold a higher proportion of their assets in the form of fixed-income securities. Stowe and Watson also found that insurers with larger surpluses tend to invest more in equities since they have a greater capacity to bear risk.

Life companies also face the additional possibility of policy loans and early surrenders on the liability side and historically this has resulted in life companies holding a much greater weight in capital-certain assets compared with pension funds. However, the introduction of financial derivatives, such as futures and options, has opened up the possibility of life companies using derivatives to hedge the risk inherent in the early exercise of these options. For example, in order to hedge against the risk of the policy loan option being exercised, the life company can sell bond futures if it expects interest rates to rise and the policy loan option to be exercised; the company may have to sell low-valued bonds to finance the loan, but is compensated for doing so from the profits made on the futures hedge. The company can subsequently buy futures when it expects interest rates to fall back again.

We have shown in this section that ALM can be fully implemented and maintained using different combinations of cash market securities. But it is possible, in principle, to use derivatives to implement ALM. The direct use of derivatives in managing portfolios has been given the name *efficient portfolio management* ("EPM"). However, for very large investors such as pension funds and life companies, it is difficult in practice to use EPM, since the derivatives markets are not sufficiently deep. Such investors are forced in practice to use the cash and repo markets to implement ALM: switching out of risky securities into cash as the value of risky securities falls below the target level and subsequently switching out of cash as security values recover.

## 6. Example: the optimal management of a defined benefit pension scheme's assets over time

In this section, we will illustrate the three-stage optimal asset allocation model using typical data for a DB scheme in the U.K. (see, e.g., Barclays Capital Equity-Gilt Study, 1999, Pension Fund Indicators, 1999, and Neill, 1977).

We will assume that an individual joins a pension scheme at age 25 years on the following terms:

Starting salary ($Y$) = £10,000 p.a.
Projected growth rate in salary ($g_Y$) = 6.5% p.a.
Years to retirement ($T$) = 40
Years of retirement ($M$) = 12
Tax rate ($\tau$) = 25%
Pension fraction at retirement ($\theta$) = 66.67%
Projected pension at retirement ($Z$) = £83,187.70 p.a.
Projected growth rate in pension ($g_Z$) = 5% p.a.
Projected inflation rate ($g_I$) = 5% p.a.
Degree of risk aversion ($\beta$) = 35 (average for the U.K., as estimated in Blake, 1996).

Financial assets with the following properties are available:

Riskfree interest rate ($r_f$) = 7% p.a.
Real interest rate ($\rho_f$) = 1.9% p.a.
Duration of equities ($D_E$) = 22.15 years.
Projected growth rate in equity returns ($g_E$) = 7.9% p.a.
Maximum available duration of index bonds ($D_I$) = 18 years
Maximum available duration of conventional bonds ($D_B$) = 14 yrs
Standard deviation of rate of change in yields ($\sigma_r$) = 0.75% p.a.
Standard deviation of rates of change in growth rates etc. ($\sigma_g$) = 0.75% p.a.
Duration risk premium ($\pi$) = 0.3% p.a.
Specific risk on equities ($\eta_E$) = 20% p.a.
Specific risk on index bonds ($\eta_I$) = 6% p.a.
Specific risk on conventional bonds ($\eta_B$) = 4% p.a.

We also assume that the appropriate discount rate for liabilities is the yield on conventional government bonds with a maturity of 15 years. Suppose that the duration of such bonds is 8.5 years, then the discount rate for liabilities ($r_L$) is 9.5% p.a. Using this data, we can equate equations (3) and (4) for $t = 0$ and solve for the required contribution rate ($\gamma$) at 12.18% of salary. This implies that at the start-up of the scheme, the present value of pension liabilities and assets (which at the start-up comprise only the projected contributions) is £59,094, while the durations of the liabilities and assets are 46.28 and 19.88 years respectively.

Figure 8 plots the present value of the liabilities and the value of the accumulated contributions during each of the 40 years' membership of the scheme. By the 40th year, they are both valued at £884,906, sufficient to provide a pension of £83,188 per year (indexed to inflation) for the next 12 years. Figure 9 plots the present value of the remaining pension

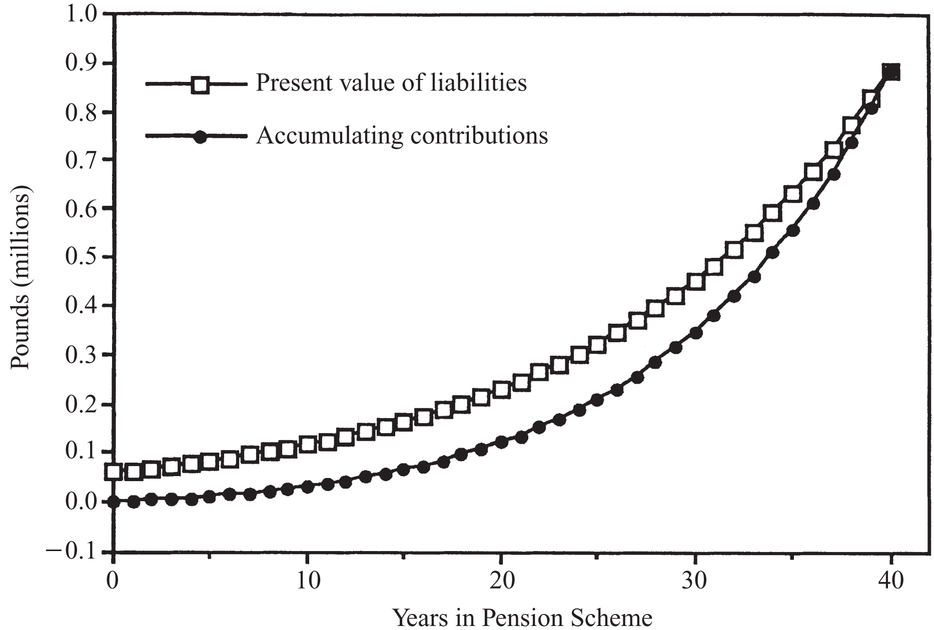


*Figure 8: Pension Liabilities and Contributions*

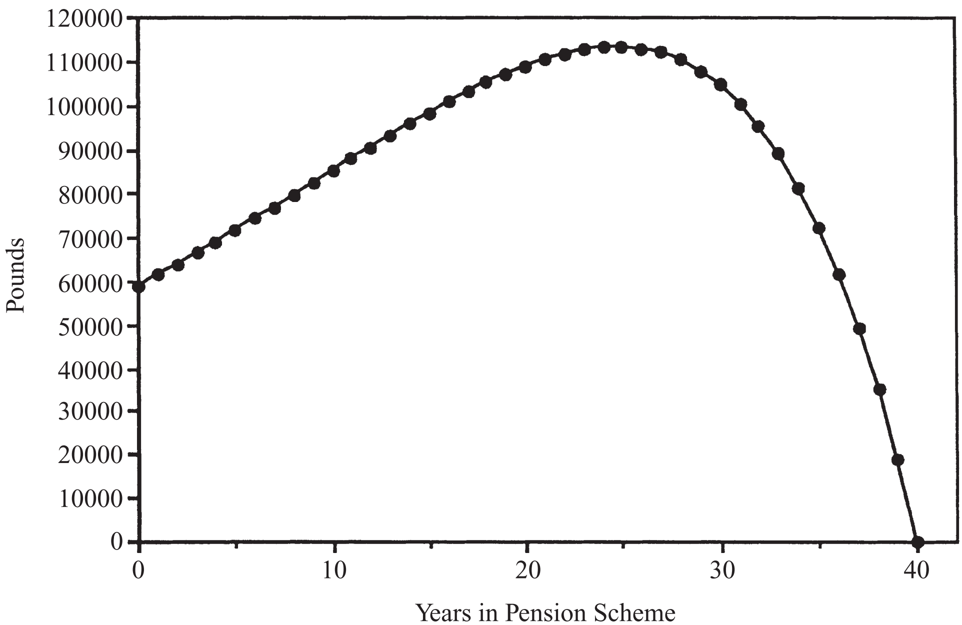


*Figure 9: Present Value of Remaining Pension Contributions*

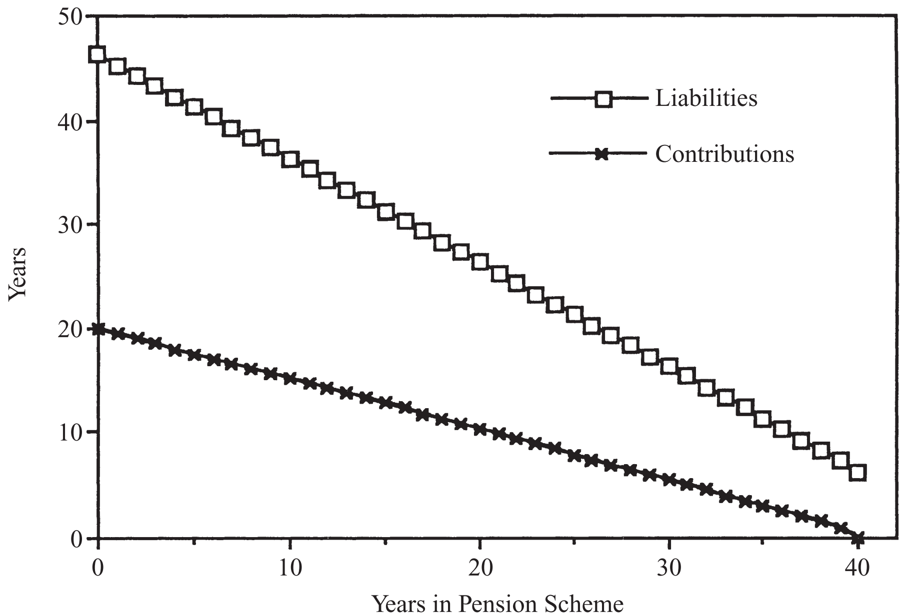


*Figure 10: Duration of Pension Liabilities and Contributions*

contributions for each year's membership of the scheme. This rises from £59,094 at the start to reach a maximum of £113,292 in the 25th year, thereafter declining rapidly to zero.

Figure 10 plots the durations of the pension liabilities and remaining pension contributions for each year of the scheme. The duration gap between liabilities and remaining contributions declines from 26.4 years at the start of the scheme to 6.3 years at the end. This gap has to be filled as rapidly as possible with financial assets but, as Figure 11 shows, the duration of financial assets required to do this is initially substantially greater than the maximum available duration of 22.15 years. Figure 12 shows how the duration gap is filled. It takes 26 years to build up sufficient financial assets with sufficient duration to eliminate the gap. This is stage 1 of the ALM strategy. The objectives of stages 2 and 3 are to build up the LIP and then to meet the reward–risk target of the member or sponsor, while preserving the duration gap at zero. Figure 13 shows the changing financial asset composition that achieves these objectives over the life of the scheme. For the 26 years of stage 1, only equities are held in the portfolio of financial assets. During the three years of stage 2 (from years 27 to 29) the equity holdings fall dramatically as index bonds take an increasingly important role in the portfolio. By year 28, the holding of index bonds exceeds that of equities; by year 30, the duration gap can be filled with index bonds of lower duration than the maximum available for index bonds. The final stage 3 lasts 11 years. It is only at this stage that the fund manager is able to take into account the reward–risk preferences of his client. For the client in this example, given his degree of risk aversion, the optimal strategy is to replace almost the entire holdings of equity with a combination of conventional bonds and index bonds.

Figure 14 shows the corresponding pattern of weights of financial assets in the

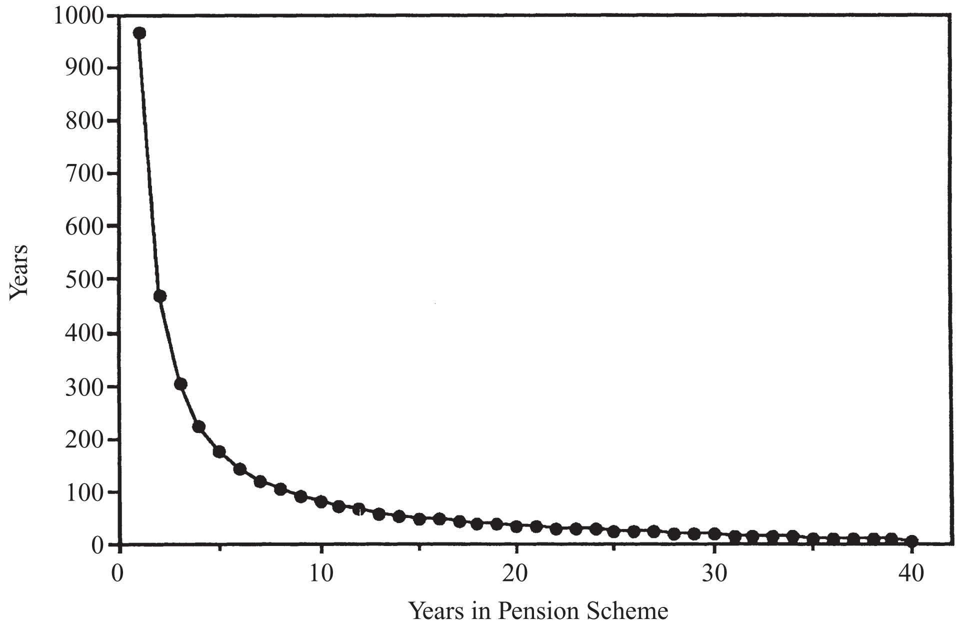


*Figure 11: Required Duration of Financial Assets*

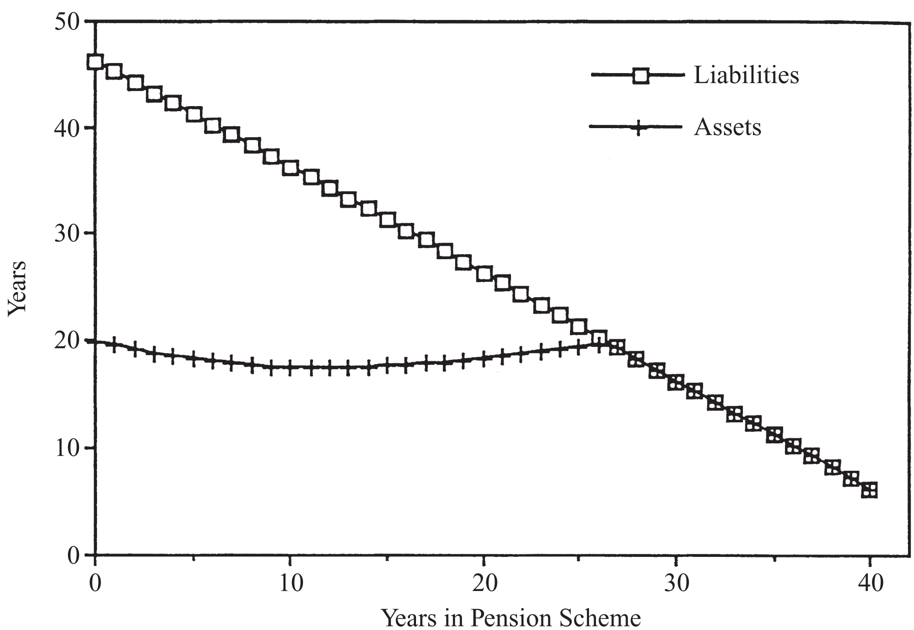


*Figure 12: Duration of Pension Liabilities and Assets*

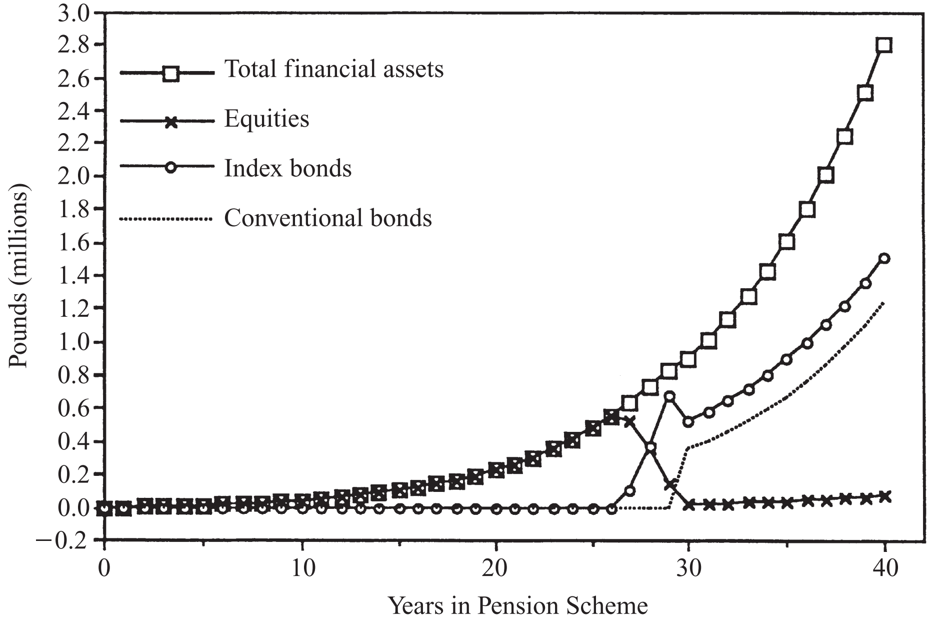


*Figure 13: Financial Asset Composition (Values)*

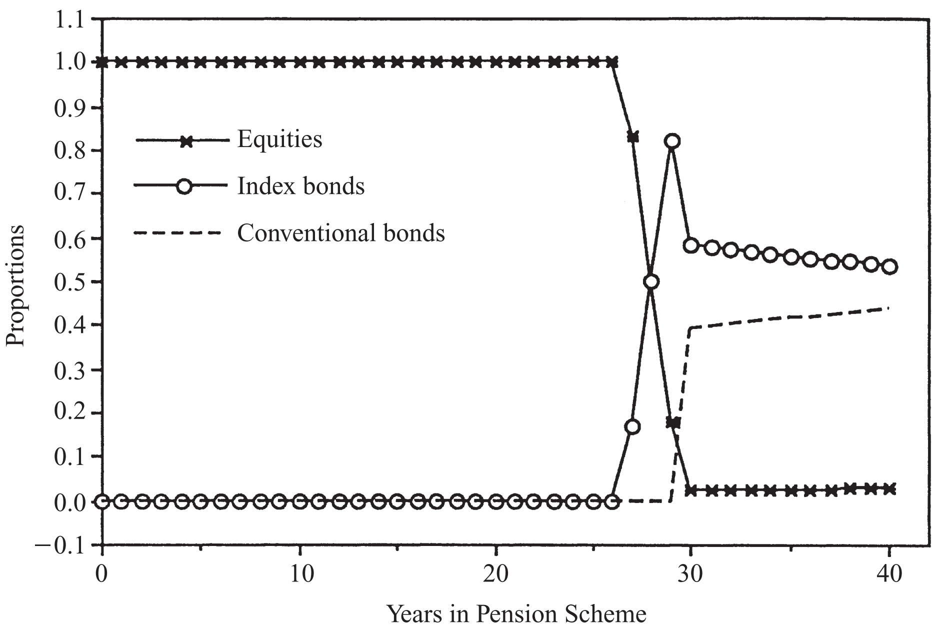


*Figure 14: Financial Asset Composition (Weights)*

portfolio necessary to achieve the overall set of objectives: during stage 3, the average holdings of equities, conventional and index bonds are respectively 2.6%, 41.5% and 55.9%. Figure 15 shows how these weights translate into portfolio expected returns and risks (as measured by standard deviations) over the life of the scheme. The three stages of the ALM strategy are clearly discernible, with the effects of increasing diversification leading to risk falling substantially more than returns over the life of the scheme.

The final set of figures examines the properties of the pension fund surplus and the consequences for the call and put options. Figure 16 shows that so long as the fund manager invests in financial assets with expected returns ($r_F$) exceeding the yield ($r_L$) at which the liabilities are discounted (which for illustrative purposes is the case in this example), the pension fund surplus (which also equals the value of the call option) will grow exponentially: in this example, the scheme will generate a surplus of over £1.9 million at maturity. In contrast, Figure 17 shows that surplus risk, as measured by the standard deviation of the surplus, declines rapidly to zero by the end of stage 1, and remains at zero for the remainder of the scheme: this is precisely the intention of ALM. The value of the put option is shown in Figure 18. Both the call and put have starting values of £36,880, about one-third of the present value of the liabilities and over 3.5 times starting salary. Thereafter, the call increases exponentially in value to reflect the pension fund surplus. In contrast, the put declines in value to zero by the end of stage 1, reflecting the zero surplus risk attached to the scheme by this stage.

Life companies would pursue precisely the same dynamic ALM strategy in the case where they had indexed liabilities. In the case where their liabilities are fixed in nominal

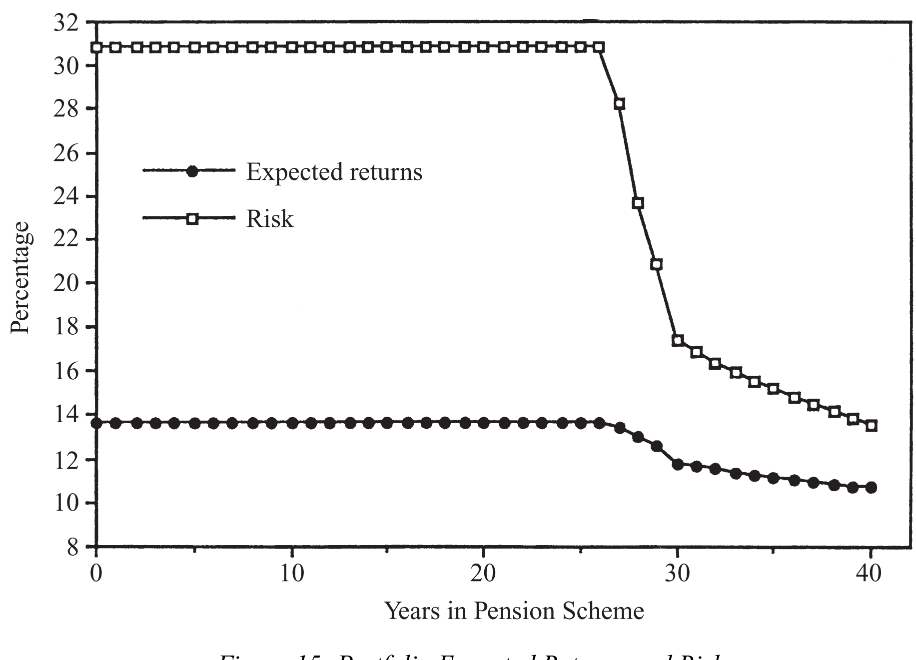


*Figure 15: Portfolio Expected Returns and Risk*

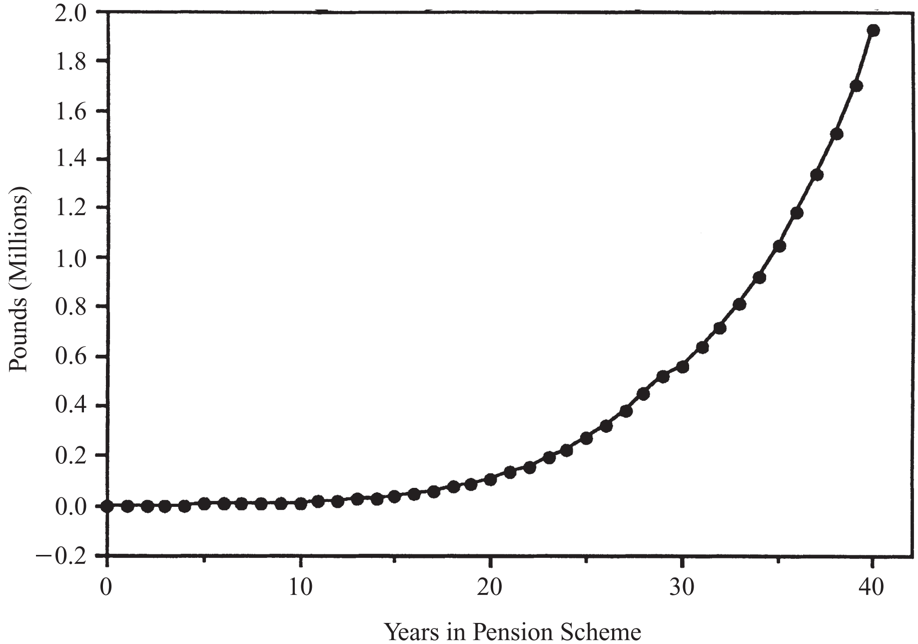


*Figure 16: Pension Fund Surplus and Value of Call Option*

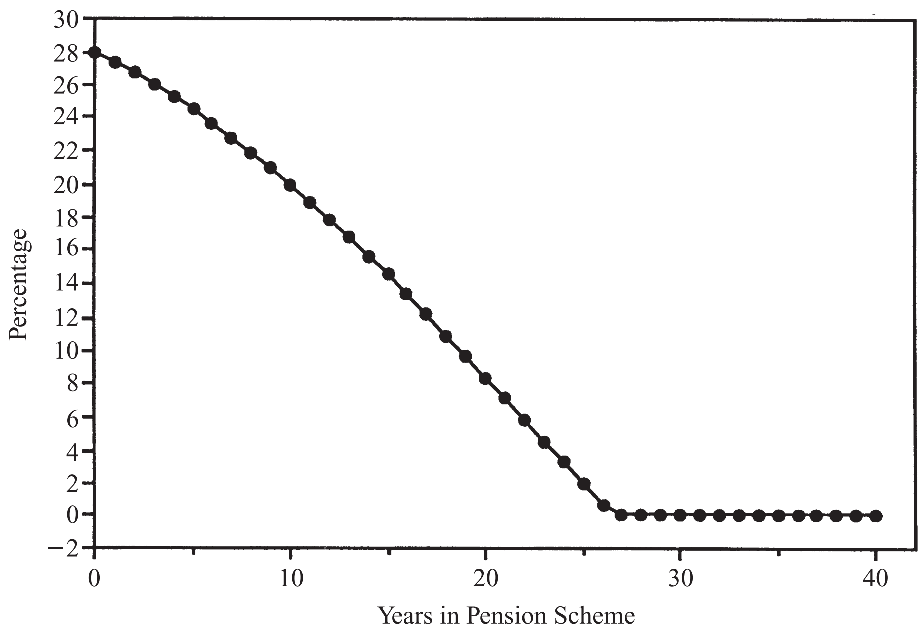


*Figure 17: Standard Deviation of Pension Fund Surplus (Surplus Risk)*

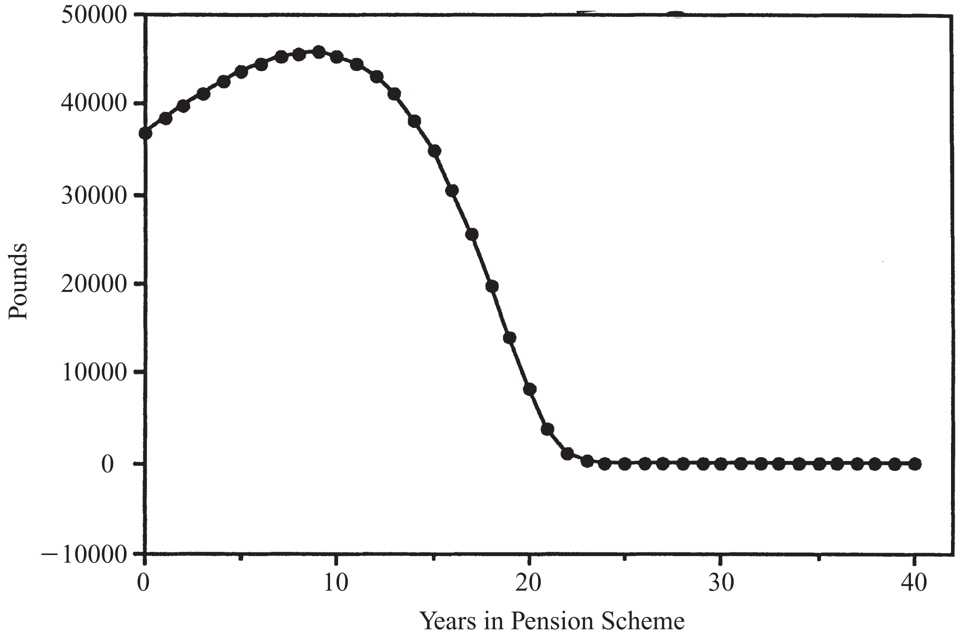


*Figure 18: Value of Put Option*

terms, the stategy would be implemented using a portfolio of conventional bonds with a duration to match the declining duration of the liabilities over time.

## 7. Conclusion

The appropriate portfolio choice model for both pension funds and life assurance companies is one based on asset–liability management, that is, managing the surplus between assets and liabilities and paying due attention to the nature of the liabilities. We showed that the liabilities of pension funds and life companies could be interpreted as particular combinations of options. The options were generally Fischer–Margrabe options, involving the exchange of assets of uncertain value, and depending on the circumstances they could be calls or puts, American or European. In the light of the option composition of the liabilities, the appropriate ALM strategy had two aspects: either the options had to be replicated or they had to be hedged against early exercise. Given the huge size of pension funds and life companies, it is clear that the only feasible procedure for replicating the options is to use combinations of cash market securities. This involved switching systematically between high and low duration securities (between equities, index bonds and conventional bonds in the case where the liabilities are indexed, and between high and low duration conventional bonds in the case where the liabilities are fixed in nominal terms), matching continuously the durations of assets and liabilities, so that the surplus is never negative and surplus risk is always zero. In the case where it is desirable to hedge against the exercise of, say, policy loan or surrender options, this could be achieved using a related financial derivative, such as bond futures. If the market in bond futures is not sufficiently deep, then a formal hedge using futures is difficult to implement, and the life company will have to keep capital-certain, if low-yielding, assets in the portfolio to meet the demand for policy loans or early surrenders.

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